
ERECTUS NON ELATUS

INTERNAL BALLISTICS.

BY

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NEW YORK: 12, CORTLANDT STREET.

1889.

To

MONS. ÉMILE SARRAU,

INGÉNIEUR-EN-CHEF DES POUDRES ET SALPÊTRES,

This Treatise

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BY

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Pag

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PREFACE.

ERRATA.

Page 69, line 9 from top, *for* " w " *read* " q."
" " " " *after* " evolved " *read* " per unit of weight."
" " " 11 from top, *for* " $P_0\, v_1\, q$ " *read* " $P_0\, v\, q$."
" " " 7 from bottom, *for* " $v = V_0$ " *read* " $v = \frac{v_0}{q}$."
" " " 6 from bottom, *for* " $10\, f\, w$ " *read* " $10\, f\, q$."
" 70, lines 5 and 9 from top and 9 from bottom, *for* " $f\, w$ " *read* " $f\, q$."
" 84, line 2 from top, *for* " P " *read* " P_1."
" 91, " 8 from top, *read* $\int_0^{\zeta} \left(\frac{d^2\, y_0}{d\, \zeta^2}\right)^a d\, \zeta$.
" 100, " 10 from top, *dele* " there."
" 125, " 14 from top, *for* " ω " *read* " w."
" 127, " 21 from top, *for* " $N^{-2}\, x^{\frac{1}{4}}$," *read* " $N^{-2}\, x^{-\frac{1}{4}}$."
" 143, " 3 from top, *after* " table " *insert* " (§ 319)."
" 174, " 12 from bottom, *for* " g " *read* " γ."
" 199, " 8 from bottom, *for* " B " *read* " β."

scattered over a number of serial publications in Fran[ce], many of which are not accessible to the general body o[f] artillerists. M. Sarrau, with that spirit of liberality which characterises the true man of science, has most kindly given me permission to make use of his investigations, and it is to

PREFACE.

In my treatise 'On the Application of Wire to the Construc- of Ordnance,' published in 1884, I touched lightly on one or two questions relating to Internal Ballistics, such as chambering, slow-burning powder, and heat imparted to the gun.

Shortly afterwards I presented a paper to the Institution of Civil Engineers on "Guns considered as Thermodynamic Machines," which was published in the 'Minutes ot Proceedings,' vol. lxxx., 1884–85; and in 1887, I printed a small pamphlet on Internal Ballistics, which, however, was only circulated among a few friends.

The subject of Internal Ballistics appears to have met with comparatively little attention in this country, and although the researches of Dr. Hutton are very valuable, they, owing to the change of conditions, are inapplicable to a great extent to the present time.

The researches of French artillerists of late years have shed a flood of light on the subject of the action of gunpowder, and especially those of M. Émile Sarrau, but these are scattered over a number of serial publications in France, many of which are not accessible to the general body of artillerists. M. Sarrau, with that spirit of liberality which characterises the true man of science, has most kindly given me permission to make use of his investigations, and it is to

this that the most valuable part of the following treatise is due.

In *Chapter I.* I have briefly treated of Explosives in general.

Chapter II. treats more particularly of Fired Gunpowder, the nature of the Products of Combustion, of Ignition and Combustion, the influence of the Form of Grain, the Temperature of Combustion, the Strength of Powder, the Loss of Temperature by the cooling action of the metal of the gun, and the Pressure and Movement of the Products of Combustion.

Chapter III. is devoted to M. Sarrau's investigations of the Formulæ for Muzzle Velocity and Maximum Pressure.

Chapter IV. contains a few remarks on the Designing of Guns, and on Pressure Curves.

Chapter V. treats of Guns as Thermodynamic Machines.

I am fully sensible of the many imperfections of the present treatise, and of my own incompetency to treat this important subject in an exhaustive manner, but I am not without hope that it may be found useful and suggestive to those who are interested in artillery questions. My object has been, to the best of my ability, to combine theoretical investigations with practical utility.

In the Report of the Royal Commission on Warlike Stores (1887), presided over by Sir James Stephen, a distinction is made between what is there called the "science of gunnery," and the "science of gun construction," and I am represented as claiming the latter as my special science. I never did anything of the kind. I certainly claimed to have a special knowledge of the subject of the application of wire to gun construction, but I did not, and could not, represent gunnery as one science and gun construction as another. What I tried to show to the Commission, but apparently failed in, was that gun construction should be conducted on,

and guided by, scientific knowledge, and that such knowledge greatly depended on these theoretical considerations.

The Commissioners appeared to doubt whether it is possible to state precisely the relation in which theory and practice ought to stand to each other, and in this they were supported by Sir Frederick Bramwell, who gave it as his opinion, that it would be dangerous to give theorists control over such a matter as the manufacture of a gun.

It is a grave error to suppose that theory and sound practice are, or can be, divergent. Hypothesis and practice may, and very often do, disagree, but theory never, unless it be a false theory. De Quincey says, "Theory is, in fact, no more than a system of laws, abstracted from experience; consequently, if any apparent contradiction should exist between them, this could only argue that the theory had been falsely or imperfectly abstracted; in which case the sensible inference would be, not a summons to forego theories, but a call for better or more enlarged theories." And Kant, in his essay "On the common saying, that such and such a thing may be true in theory, but does not hold good in practice," says, "It is far more tolerable that an unlearned person should represent theory as superfluous for the purpose of his imaginary practice, than that a shallow refiner, whilst conceding the value of theory for speculative and scholastic uses, should couple with this concession the doctrine, that in practice the case is otherwise; and that upon coming out of the schools into the world, a man will be made sensible of having pursued mere philosophic dreams. In short, that what sounds well in theory is not merely superfluous, but absolutely false for practice. Now the practical engineer who should express himself in such terms upon the science of mechanics, or the artillery officer who should say of the doctrine of projectiles, that the theory of it was conceived indeed with great sub-

tilty, but was of little practical value, because in the actual experience of the art it was found that the experimental results did not conform to the theory, would expose themselves to derision. For, supposing that in the first case should be superadded to the Theory of Mechanics that of Friction, and that in the second, to the Theory of Projectiles were superadded that of the resistance of the air—which in effect amounts to this, that if, instead of rejecting theory, still more theory were added—in that case the results of the abstract doctrine and of the experimental practice would coincide in every respect."

My object has been to assist in removing the incubus of empiricism from artillery science, and whilst fully conscious of the imperfection of my efforts, and of the opportunity I have given to adverse criticism, I will only say to my critics, "Si quid rectius novisti, candide imperti."

J. A. LONGRIDGE.

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INTERNAL BALLISTICS.

CHAPTER I.

ON EXPLOSIVE SUBSTANCES IN GENERAL.

1. By the term explosive substance is meant a substance composed of two or more elements mixed together or chemically united, and such as, that when this affinity is disturbed, a violent reaction takes place, giving rise to a great development of heat, and to various new compounds in a liquid or gaseous form.

In the latter case their gaseous products expanded by the developed heat constitute a reservoir of energy which is applicable to mechanical uses.

2. The reaction varies in rapidity according to the nature of the compound substances.

In some cases, such as fulminates, nitroglycerine, &c., the reaction is extremely rapid, and is called "detonation." In others, such as ordinary gunpowder, it is much less rapid, and is called explosion; whilst in others, such as fuse or rocket composition, it is still slower, and is called combustion.

This distinction of terms, for what is in truth only one phenomenon, is at once unscientific and misleading, and it has given rise to erroneous conceptions of the action of gunpowder, which will presently be considered.

3. If by "detonation" it be said that, an instantaneous reaction is meant, the reply is, that no such thing as an

instantaneous reaction exists. "Detonation," so called, is only a very rapid "explosion," and "explosion" is only a very rapid "combustion."

The most rapid decomposition which takes place with fulminates or nitroglycerine, is a gradual process, and the attendant rise of pressure in a close vessel, owing to the evolution of heat and gas, is a gradual, and not an instantaneous rise. The phenomenon is one and the same, the only difference being in degree, and not in kind, and therefore it is desirable to designate it by one term only, that of combustion.

4. The distinction of terms has given rise to curiously erroneous ideas as to the action of gunpowder in a gun. We hear of the distinction between the "percussive" effect and the "static" effect of a charge of powder.

For instance Mr. Lynal Thomas * says, "So far from the action of the fired powder in a gun being that of a constant pressure estimated at so many atmospheres, it is violently percussive and variable to an indefinite extent, the percussive action starting the projectile with a finite velocity," and he adds, that a distinguished mathematician who witnessed some of his experiments drew up a formula in accordance with the "percussive" theory, which formula he gives as follows:—

$$v^2 = V^2 + \frac{\pi g m n' a d^2}{w} \log \frac{b}{a},$$

V being the velocity with which the shot *begins to move.*

It is hardly necessary to insist on the absurdity of a body *beginning* to move with a finite velocity.

Again, Colonel Hope † speaks of "the *percussive* effect of a charge of powder," and "the *static* equivalent of *percussive* force."

* "Action of Fired Gunpowder," by Lynal Thomas, 'Illustrated Naval and Military Magazine,' vol. i., 1884.

† 'A Revolution in the Science of Gunnery,' read at the R.U.S. Inst., 23rd July, 1884.

General Boxer, who at the time held a high position at Woolwich, during the discussions which took place in 1860 at the Institution of Civil Engineers on my paper on the Construction of Artillery said,* "In considering the force of gunpowder, there were two main points to be regarded. First, there was a certain definite pressure which might be termed the *statical force*, or the force which was effectual in giving motion to the projectile in the bore of the gun. Secondly, there was a *percussive force*, in addition to that already mentioned, and *of a different nature to it*, which principally tended to cause destruction to the material of the gun."

This so-called *percussive* force has no real existence. It is a misnomer, and only leads to confusion of thought. A finite velocity instantaneously set up would require an infinite force instantaneously applied. Such a force is of course impossible.

The chemical action of the most rapid fulminate is effected in a finite time, however small that time may be. Thus the force of an explosion is always relative. The chemical reaction of a fulminate or of nitroglycerine is very rapid compared with that of gunpowder, that of gunpowder very rapid compared with fuse composition, but in every case the reaction takes place in time, and the corresponding pressure is more or less great, according to the nature of the explosive, and the conditions of its use.

5. The mechanical effect of an explosion depends very greatly on the velocity of the chemical reaction. Berthelot states, that with dynamite and guncotton the reaction may be propagated throughout the mass with a velocity of over 20,000 feet per second. In such a case the generation of gas is almost instantaneous and the vis viva of the products striking against any object is so great as to shatter that object, even where there is no tamping.

On the other hand, the ballistic effect of such explosions is

* 'Minutes of Proceedings Inst. of Civil Engineers,' vol. xviii., 1860.

very small, for the reason that the volume of gas produced and the units of heat generated are both small.

6. In order to determine the relative force of explosives, M. Roux made a series of experiments by firing a given quantity in an absolutely closed space within a large block of lead, and he estimated the relative force of the explosion by the size of the cavity formed by the explosion. In this way he found the following results of the comparative explosive force:—

Black gunpowder	1
Picrate of potassa	5
Guncotton	7·5
Nitroglycerine	10

and for a mixture of explosives he uses the following rule:—

"Add the forces of each compound multiplied by the fraction representing its proportion in the mixture."

If, however, this rule be applied to a mixture of ordinary gunpowder and nitroglycerine, the result will be found much less than the actual result from experiment. From this it appears that the pressure of the more rapid explosion of the nitroglycerine has increased the force of the black powder, and from this it has been concluded, that for black powder two orders of explosion may exist. The first order being that of detonation; the second, that of ordinary combustion; and it is stated by French authorities that all explosions are susceptible of these two orders of explosion according to the circumstances under which they are fired.

Not that an absolute distinction can be shown in each particular case, but that those orders are the limits between which practical results may take place.

In this way the force of black powder may vary from 1 to 3·3.

Captain Roulin, of the French Artillery, gives the following table of results showing the relative force of the two orders of explosion in different explosives.

Explosive.	1st order.	2nd order.
Black powder	4·34	1·00
Nitroglycerine	10·13	4·80
Guncotton	6·46	3·00
Picrate of potassa	5·31	1·82
Fulminate of mercury	9·28	..

The order of the explosion which will take place depends upon external conditions of firing.

Dynamite and guncotton may be burnt in the open air without any explosion by the simple contact of flame; on the other hand, if the gases are confined by an envelope of more or less resistance, or if the material be previously heated to a certain degree, there will be an explosion of the 2nd order of more or less violence.

If, however, the explosion is effected by means of a detonator, such as fulminate of mercury, this will give rise to a detonation or explosion of the first order.

7. It is evident, therefore, that the pressure existing in the barrel of a gun with any particular powder may be greatly affected by the circumstances under which the ignition takes place, and the subject has much interest as regards the so-called "wave pressures" in guns.

M. Berthelot treats of this subject in his treatise 'Sur la force des matières explosives d'après la Thermochimie,' 3rd ed., 1883.

According to his views every explosive reaction must be referred to an initial rise of temperature, which may be caused by ignition, by a shock, or by friction, and this reaction is transmitted from particle to particle successively. Certain explosive materials decompose spontaneously and slowly under ordinary temperatures, without explosion, whilst their detonation takes place when the temperature is suddenly and largely increased either purposely or by accident.

M. Berthelot distinguishes the propagation of the reaction by two classes:—

1st, That of combustion.

2nd, That of detonation.

Between these two classes there may exist a series of intermediate modes of explosion. In fact, the passage from one class to another is accompanied by violent and irregular movements of the material, during which the propagation of the combustion acts by a vibratory movement of increasing amplitude and with more or less velocity.

Class of Combustion.

8. When an explosive material is gradually heated to a sufficient degree, a portion of it explodes; if the gases are free to expand, the pressure rises slowly and a fresh portion of the material is ignited, and thus the inflammation is propagated from particle to particle with a velocity dependent on the circumstances of the case. Such is generally the course of action with ordinary gunpowder.

Class of Detonation.

9. When a shock sufficiently violent is produced in one part of an explosive substance, and if the pressures which result from this shock are too sudden to be propagated to the whole mass, the transformation of the vis viva into heat will take place chiefly in the first portion of the mass. This may thus be raised to a sufficient temperature to detonate. If the first production of gas is so rapid that the mass of the material has not time to be displaced, and if the expansion of the gas produces a more and more violent shock on the adjoining portion of the material, the vis viva of this new shock will be transformed into heat, and thus give rise to the detonation of a new portion of the material. This alternate action of a shock the vis viva of which is transformed into heat, and a production of heat which raises the temperature of the next portion so as to produce a new

detonation, transmits the reaction from portion to portion throughout the entire mass.

The propagation of the inflammation then in this class of detonation may be compared to that of a wave of sound, that is to say, it is a trüe wave of explosion travelling with a velocity incomparably greater than that of a simple ignition transmitted by contact from particle to particle, and when the gases freely expand as they are produced. It must also be remarked that whilst the wave of sound is generated by a periodic succession of similar waves, that of explosion is not periodic, but takes place once for all.

10. An explosion of the second order may be transformed into one of the first.

The velocity of propagation of reaction in a case of the second order, is greater as the molecular intensity of reaction is greater, this being defined by the quantity of material transformed into gas at a fixed temperature and under constant pressure.

It increases also, (1) with an increase of the initial temperature of the mass.

(2) With the increase of the weight of the charge, because in this case the influence of cooling is proportionately less.

(3) With the increase of pressure under which the gas is generated.

When the explosive matter is confined by a tamping, the pressure will rise very rapidly, and the velocity of propagation may be sufficiently great to give rise to a pressure or a shock capable of detonating a portion of the mass.

This is no doubt the case in long charges of small grained powder ignited at the rear. The forward portion of the charge is jammed up against the projectile, and the reaction is converted from one of the second to one of the first order, giving rise to the so-called local "wave pressures" which have been observed.

In operating with an explosive of which the molecular velocity of reaction is very great, such, for instance, as nitro-

glycerine or fulminate of mercury, no tamping is required; the gases are developed so rapidly that the environment, be it solid, liquid, or even gaseous, has not time to be displaced, and opposes itself like a fixed wall to the action of the explosive during the infinitesimally small time of reaction.

Of the Potential of an Explosive.

11. By the "Potential" of an explosive is meant the mechanical equivalent of the heat given out by its combustion.

Thus if E be the mechanical equivalent of heat, and Q the units of heat developed by the combustion of unit of weight, E Q is the "Potential." This Potential is independent of the manner in which the combustion takes place, provided that it be complete and the final state of the products the same.

Making use of French unities, and the French equivalent of heat, or E = 436, Messrs. Sarrau and Roux determined the Potentials of several explosives as given in the following table:—

POTENTIALS OF EXPLOSIVES.

Denominations.	Potential. Metric tons.	Potential. Foot tons.	Composition.		
			Saltpetre.	Sulphur.	Carbon.
French.	per kilogramme.	per lb.			
Fine sporting powder	370	542	78	10	12
Ordinary common powder ..	347	508	75	12·5	12·5
Rifle powder, B	337	494	74	10·5	15·5
Mining powder	267	391	62	20	18
Nitroglycerine	778	1140			
Guncotton	489	716			
Picrate of potassa	366	536			
Noble and Abel.					
Cocoa	365	535			
Spanish pellet	335	490	75·6	12·5	11·5
Curtis and Harvey, No. 6 ..	333	488	74·7	10·4	14·0
F.G., Waltham Abbey	322	471	74·	10·1	15·
R.L.G. "	317	464	74·6	10·1	14·3
Pebble "	314	460	74·8	10·1	14·2
Mining	225	330	62	15·2	21·4

12. The Potential of an explosive must not be confounded with the mechanical effect which may be obtained from it; neither must it be confounded with the pressure developed by exploding it in a close vessel.

In treating hereafter of the action of gunpowder these differences will be fully explained.

13. As regards explosives, a distinction may be made between two classes.

First, mechanical explosives, in which the reagents are uncombined, but intimately mixed.

Second, chemical explosives in which the ingredients are united by a more or less unstable affinity.

In the first case, the particles which have to combine are separated by appreciable distances, whilst in the second, each molecule contains within itself the reagents, and is in fact a complete explosive *per se.*

The reaction in the second class is therefore much more rapid, and the *violence* of the explosive proportionately great. Moreover, in many of the second class, the chemical union is exceedingly unstable, and consequently more subject to the influence of external action.

Of the first class, by far the most important is gunpowder, composed of nitrate of potassa, sulphur, and charcoal, the usual proportions in this country being 75, 10, and 15 respectively, and the same proportion, or very nearly so, has of late years been adopted by France and Belgium for all large guns.

An exception must, however, be made with regard to "cocoa" powder, the composition of which is kept secret. It differs from the ordinary powders in the nature of the charcoal, which is made from rye straw instead of the dogwood and alder generally used, and in the small proportion of sulphur.

14. The composition and manufacture of explosives generally, is foreign to my present purpose, as gunpowder is now exclusively used for artillery purposes. Such powder is of the description commonly called nitrate powder, the oxidising agent being nitrate of potassa.

There are, however, other descriptions of powder which may be briefly mentioned, viz.:—

15. (1) Nitrate of Soda Powder.—If for nitrate of potassa, nitrate of soda be substituted in the same proportions, a powder is obtained which is somewhat less costly and about one-third stronger than the nitrate of potassa powder.

It is, however, more subject to absorb moisture, although this is probably due to the imperfect fabrication of the nitrate of soda, which when pure is not deliquescent, and probably only acquires that property by the presence of deliquescent salts, such as chlorides.

Nitrate of soda powder was used to a very great extent in the excavation of the Suez Canal.

16. (2) Nitrate of Baryta Powder.—This powder is of a slower combustion than nitrate of potassa powder, and consequently strains the gun less severely. It has, however, the property of fouling the gun to a considerably greater extent.

17. (3) Chlorate of Potassa Powder.—With equal weight this salt contains less oxygen than nitrate of potassa, but it decomposes more readily and more completely, consequently this powder is more violent. It is also somewhat dangerous, being liable to explode under a sudden shock, and is more erosive to the iron and offensive to the service from the chlorine gas which is evolved.

18. (4) Picrate of Potassa Powder.—Powder combined of picrate of potassa, saltpetre, and charcoal was tried by M. Designolle in France in 1867.

This powder gave good results in torpedoes and also in guns, and had the advantage of not evolving either sulphurous acid or sulphide of potassium, being so far of considerable advantage in ships and casemates, but several cases of premature explosion having taken place, the use of the powder does not appear to have been persevered in. It is, moreover, costly, and the manipulation of picrate of potassa is somewhat dangerous.

19. (5) Picrate of Ammonia Powder.—This salt is less sensible to a sudden shock than the picrate of potassa. In

the open air it burns like a resinous matter, giving out a dense smoke, but in a close vessel the combustion may become very rapid and the force violent.

A powder, consisting of 54 parts of picrate of ammonia, and 46 parts of saltpetre, was tried at Calais, but found too violent in its action for artillery practice.

20. Although hitherto Picrate powders have met with little favour, it is not improbable that a further study of the subject may remove the objections which have been made, and it is very desirable that this should be undertaken, as these powders are nearly smokeless, do not foul the gun nor erode the bore to the same extent as ordinary powder.

I believe that experiments with these powders are still being carried out in France.

On the Physical Characteristics of Gunpowder.

21. The chief physical features of gunpowder are :—

(1) The absolute density.

(2) The gravimetric density.

(3) The form and dimensions of the grain.

(4) Its hydroscopic quality, or its capacity for absorbing moisture.

Absolute Density.

22. By the term "absolute density" is denoted the weight of unit of volume of the powder compared with the weight of the sâme volume of water at a given fixed temperature.

In France, since the kilogramme or unit of weight, is the weight of a litre or unit of volume of water, it is obvious that the absolute density is the same as the absolute weight of unit of volume, an advantage which the English system does not permit of.

The absolute density of nitrate gunpowder varies according to the process of manufacture from 1·50 to 1·90.

Gravimetric Density.

23. This term has different meanings, in England and in France.

24. In England, it is defined in the Text-Book of Gunnery* to be "the ratio of the weight of a charge of powder in the chamber of a gun, to the weight of that volume of water which would fill the space behind the projectile."

Thus if gravimetric density = 1, the space occupied by 1 lb. of the charge is the same as would be occupied by 1 lb. of water, i. e. 27·73 cubic inches, or in other words, if the charges be spaced so as to allow 27·73 cubic inches to the pound of charges, the gravimetric density is unity.

For any other spacing if n be the number of cubic inches allowed to the pound of charge,

$$\text{Gravimetric density} = \frac{27\cdot73}{n}.$$

The usual notation in England for denoting the condition of a charge is to write first the weight of the charge, then the designating mark of the powder, and lastly the quotient of 27·73 by the number of cubic inches allowed to the pound. Thus 75 P$\frac{27\cdot73}{30}$ signifies 75 lb. of P powder spaced at 30 cubic inches per lb.

25. In France, this is called "Densité de Chargement," as distinct from "Densité Gravimétrique," by which latter term is meant, the weight in kilogrammes of 1 cubic litre of the powder not pressed together, except by its own weight.

Consequently Densité Gravimétrique = 1, means 1 kilogramme occupying a space of 1 litre or 1 decimetre cube, or in our units 2·2 lb. of powder in a space of 61·02 cubic inches, or 1 lb. of powder in a space of 27·73 cubic inches, which is the same as in our system.

But it is to be noted, that one decimetre cube of powder does not always weigh 1 kilogramme. The weight increases

* 'Text book of Gunnery,' by Major S. Mackinlay, p. 22, 1887.

with the size of the grain, so that whilst with the small-grained powder designated F_2 in France, it is 934 to 944 grammes with the large $A\frac{30}{40}$ it is 1150 grammes, and the relative "densités gravimétriques" are ·934 and 1·150.

This distinction is not made in England, and our "gravimetric density" is equivalent, not to "Densité Gravimétrique," but to "Densité de Chargement."

Form and Dimensions of Grain.

26. As will be seen hereafter, the form and size of grain have very important effects upon the action of powder in a gun.

A few years ago the powder called R.L.G. was the only powder used for artillery purposes.

Owing to the small size of the grain, the time of combustion was exceedingly small, and the action violent. At the same time, owing to the small space between the grains, the ignition was slow and very irregular, and this, as will be shown presently, gave rise to abnormal pressure, called, *faute de mieux*, "wave" pressure by our artillerists.

By degrees, and by a sort of tentative process, the size of the grains was increased, though the form remained more or less irregular. Such were the P and P_2 powders.

Subsequently moulded grains of a regular form were introduced, such as the spherical, cubical, cylindrical, and finally the prismatic, and quite recently Mr. Quick has introduced the form of cylindrical discs placed one on the top of the other, perforated with cylindrical holes and with radiating or other channels on the face of the discs so as to facilitate ignition.

The influence of these different forms will be examined hereafter.

Hydroscopic Quality.

27. All gunpowder is more or less liable to absorb moisture and thus to deteriorate in strength by long storage, giving rise to corresponding irregularity in ballistic results,

but even when first manufactured there is considerable difference in the amount of moisture in different powders.

The following table gives the percentage of water in various powders as determined by Noble and Abel in England, and MM. Sarrau and Roux in France :—

English	Name of Powder.	Cocoa.	P.	R.L.G.	F.G.	Spanish.	Curtis and Harvey 6.	Mining.
	Percentage of water.	1·33	0·95	1·06	1·48	0·65	1·17	1·61
French	Name.	B rifle.	SP_1 SP_2 SP_3	C_2				
	Percentage of water.	1·10	1·2 to 1·8	1.3 to 1·7				

CHAPTER II.

FIRED GUNPOWDER.

Products of Combustion.

28. When a charge of powder is fired, about 43 per cent. in weight is converted into permanent gases, that is to say, into gases which when cooled down to ordinary temperatures retain their gaseous condition.

The remainder, or about 57 per cent. by weight, is, when cooled down, a solid residue, but whilst in the gun at the high temperature of ignition, is in a liquid form diffused through the gases. When the charge is fired in a close vessel and the gases are afterwards allowed to escape, this liquid rapidly solidifies.

29. The chemical composition of the products of combustion has not much interest in a ballistic point of view, and the compounds, especially the solids, appear to vary within very considerable limits. Those who are interested in this question will find it very ably discussed by Messrs. Noble and Abel in their papers published in the Transactions of the Royal Society in 1875 and 1879, and also in a report by MM. Morin and Berthelot in the Académie des Sciences ('Comptes Rendus,' lxxxii.).

The solids consist chiefly of compounds of potassium, with carbonic, sulphuric, sulphurous, and nitric acids.

The gases are approximately represented as follows:—

Carbonic acid	·2596
Carbonic oxide	·0343
Nitrogen	·1084
Sulphurous acid	·0099
Marsh gas	·0003
Hydrogen	·0007
Oxygen	·0003
Water	·0148
	0·4283 per cent.

30. By some it is believed that, at the very high temperature prevailing in a gun, the liquid residue is itself converted into the gaseous form, and that it gives out work during its expansion, but it is maintained by Noble and Abel, that the work done by expansion is due only to the gaseous portion of the products, although these are sustained during their expansion by heat imparted to them by contact with the particles of liquid diffused throughout their volume at a very high temperature. This hypothesis will be discussed further on.

At present it may be added, that even if the 57 per cent. of solids be in a gaseous form, it has been shown by Schischkoff and Bunsen that the tension of such gases must be exceedingly small, and quite incapable of producing any appreciable effect on the general tension of the permanent gases.

31. It is shown by Noble and Abel that at the temperature of ignition, the volume of the 57 per cent. of inert liquid is approximately equal to that of the powder from which it is derived.

Dissociation.

32. By some authorities it is maintained, that although the various compounds determined by analysis are found in the final state of the products, they do not exist in the earlier stages whilst the action is taking place in the projectile.

They say that these compounds cannot exist at the high temperature there reigning, owing to the interference of the phenomenon called "dissociation," but can only arise when the temperature has so far fallen as to admit of such secondary combinations.

33. Although there is no positive evidence of "Dissociation" in a gun, it may be well to examine what would be the effect ballistically, if it did take place.

34. It appears from the table (§ 29) that about 60 per cent. of the gaseous products consist of carbonic acid; as will be shown hereafter, the temperature of ignition is about 2340° C., and as the temperature of dissociation of carbonic acid is about 1800° C., it is held that the carbonic acid could not exist until the temperature had fallen to 1800° C., and that up to this time only carbonic oxide could exist. But the question may well be asked, why should not the carbonic oxide be subject to dissociation as well as the carbonic acid? If this were so, the gaseous products of combustion would be confined to the oxygen liberated by the decomposition of the nitrate of potash, and of course this would only take place at a lower temperature than 1800° C.

Consequently, even if the whole of the carbon were converted into carbonic oxide, we would have the volume of this gas and the volume of the remaining oxygen existing together so long as the temperature exceeded 1800°, and this would give a comparatively low pressure, until by the fall of temperature, the combination of the free oxygen with the carbonic oxide would give rise to a sudden and very large increase of temperature, and a sudden almost explosive increase of pressure.

The result would therefore be a low initial pressure, decreasing as the shot travelled towards the muzzle, and then a very sudden rise of pressure, again falling till the shot left the gun. This certainly does not represent the real conditions.

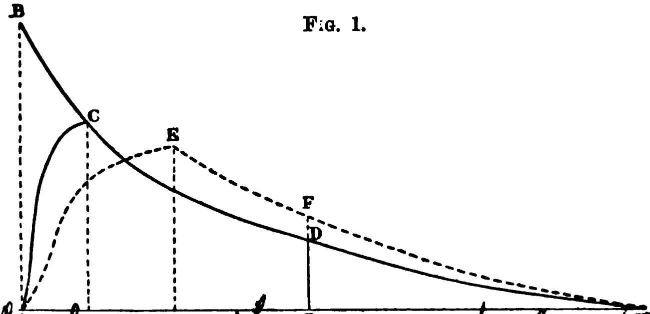
35. It is true that the volume of one equivalent of oxygen plus that of one equivalent of carbonic oxide is one and a-half times greater than that of the resulting equivalent of carbonic acid, and this would increase the pressure as long as dissociation took place, but the sudden increase of pressure when the temperature fell to 1800° would equally take place.

36. But it may further be remarked, that it by no means follows, that because carbonic acid may be dissociated at a temperature exceeding 1800°, under atmospheric pressure,

the same would take place under the enormous pressures existing in a gun.

37. Another argument against dissociation may be thus stated.

FIG. 1.



If the charge is the same the two curves of adiabatic expansion coincide -

If a charge of powder be fired in a gun, the length of which is represented by A G, and the projectile be not allowed to move until the whole of the charge is burnt, the pressure may be represented by a curve B C D which is Noble and Abel's curve, and if the gun be sufficiently lengthened, the pressure would fall to zero at a point X on the same curve.

If, however, the projectile be allowed to move, the pressure curve will be an ascending curve up to the point of maximum pressure C when all the powder is burnt, after which it will descend as before to X.

The work done on the projectile will, therefore, be represented by the area A C D G.

Now if from dissociation or otherwise, the point of maximum pressure be removed further forward to E, the pressure curve will be A E F X falling to zero at the same point X, and the work done on the projectile will be A E F G.

Now the area A C D X must be equal to A E F X because the whole of the heat is expended in the two cases. But F G X is greater than D G X, therefore A C D G must be greater than A E F G.

It therefore is evident that dissociation must decrease the ballistic effect as represented by Noble and Abel's curve, but as this curve fairly represents the results obtained in practice the conclusion is, that dissociation does not really take place in a gun.

38. It may here be remarked that the reasoning in the last paragraph equally applies to the decreased ballistic effect of a slow as compared with a quick powder, and therefore that the loss of effect with the former, must be made up by increased weight of charge.

39. For the above reasons I am of opinion that dissociation does not practically take place in the combustion of powder in a gun.

Ignition.

40. It is important to distinguish between Ignition and Combustion.

By Ignition is meant (*a*) the commencement of chemical action caused by contact with some source of high temperature, and (*b*) the communication of this action from the portion of the surface where it commences to adjacent portions and from the surface of one grain of powder to that of others.

41. When a train of powder is fired in the open air, the velocity of transmission increases as the volume of powder per unit of length increases.* The following table shows the results obtained by General Piobert with cannon powder.

Formation of Train.	Weight of powder per lineal foot. lb.	Velocity in feet per sec.
Single grains in contact, 82 per lineal foot	·00282	1·214
Grains superimposed	·02016	5·577
" "	·04032	7·613
" "	·08046	9·646

* Piobert, 'Traité d'Artillerie,' 1847, p. 164.

42. Piobert further found, that if a train of powder, instead of being simply laid on a flat surface, is laid in a groove and covered with a light plank, the velocity of transmission is about doubled when the groove is entirely filled with powder.

If, however, it is only half filled, the velocity is still further increased by about one-half.

On firing trains in tubes he found analogous results, as is shown in the following table :—

Formation of Train.	Weight of powder per lineal foot. lb.	Velocity of transmission in tube. feet per sec.	Velocity of same train in open air.
Tube 0·512 in. diam., 1 thickness of paper, full	·08064	5·118	9·646
" " 16 " full	·08064	12·137	9·646
" " 16 " half full	·04368	25·591	7·776
" 0·785 " 16 " full	·21168	13·780	..

43. From his very numerous and carefully conducted experiments, Piobert arrived at the following conclusions :—

(*a*) The velocity of Ignition varies very nearly in the inverse ratio of the fourth root of the diameter, or equivalent diameter, of the grain.

(*b*) It decreases with the increase of density and the degrees of glazing.

(*c*) The composition (*dosage*) and length of trituration of the powder do not appear to have any appreciable effect on the velocity of Ignition.

Combustion.

44. By the Combustion of a grain is meant, the gradual burning downwards from the surface until the whole is consumed.

45. On this subject Piobert also made many experiments and concluded as follows:—

(*a*) That *cæteris paribus*, the velocity of Combustion varies inversely as the density of the grain, or $v\,\delta = a$, a constant which depends upon the nature of the powder, and denotes the weight of powder burnt per second per unit of surface.

(*b*) The velocity decreases rapidly with an increase of humidity in the powder.

(*c*) It increases with the amount of trituration up to a certain limit.

(*d*) With the same density, same trituration, and same humidity, the greatest velocity of Combustion was obtained from a powder composed of

Saltpetre	75 parts.
Carbon	15 „
Sulphur	10 „

(*e*) In free air, the velocity of Combustion varied, in the different powders used by Piobert, from 0·4 inch per second to 0·6 inch per second.

46. When powder burns under pressure, as in a gun or close vessel, the velocity of Combustion is very greatly increased. After careful examination of the results obtained by himself and other artillerists, Mons. Sarrau adopts the following formula:—

$$V = V_0 \left(\frac{p}{p_0}\right)^{\frac{1}{2}} \; . \; . \; . \; . \; . \; .$$

where

V = velocity of Combustion at pressure p.
V_0 = „ „ atmospheric pressure p_0.

That is to say, that the velocity varies as the square root of the pressure.

Thus, a powder which in the open air would burn with a velocity of 0·6 inch per second, would burn with a velocity of about 33 inches per second under a pressure of 3000 atmospheres.

Effect of Rate of Ignition and Combustion on Pressure.

47. From what precedes, it is obvious that the two phenomena of Ignition and Combustion have each a distinct part in the conversion of a charge of powder into its products, and that it is on their joint actions that the effect of firing the charge depends.

48. When a charge of powder, placed behind a projectile, is ignited at the rear end, the Ignition commences with the grains at that end, and gradually extends throughout the charge. If the rate of Combustion be great and the size of the grains small, it may be, that the grains at the back are entirely consumed before those in front are ignited, and this is likely to take place with a small-grained powder closely packed in a long tube, that is to say, in a long charge of fine-grained powder.

This rapid development of gas at the rear end of the charge would give rise to a considerable local pressure and compress the powder next the projectile, wedging it up as it were into a mass of high gravimetric density, before the projectile had moved to any considerable extent.

On the other hand, if the cartridge did not quite fill the chamber, the Ignition would pass rapidly through the vacant space, and the Combustion would take place almost simultaneously at both ends of the cartridge. The result of this would be the early displacement of the projectile and an increase of space for the evolving gases, giving rise to a lower pressure behind the projectile.

The same effect would take place if a large-grained powder were used, as in this case the interstices between the grains being large, the Ignition would pass rapidly, and the early action of the pressure on the base of the projectile would move it forward and increase the space.

49. Rapidity of Ignition throughout the charge is therefore a matter of considerable importance as preventing local variations of pressure.

It may be promoted in various ways, viz. by increasing

the size of the grains, by perforations through the mass of each grain, as in the case of the cylindric and prismatic powders, by leaving a space between the cartridge and the top of the chamber, by commencing the ignition at the centre of the charge, or by igniting it simultaneously at several places. Of course the more simultaneous the Ignition, the more rapid is the development of gas, and the more rapid the rise of pressure.

50. The rate of Combustion has a greater effect as regards the pressure than the rate of Ignition. It is on it that the rate of evolution mainly depends, but the form of the grain has also a very important effect, and it is the rate of evolution of the gas which chiefly affects the pressure.

51. The rate of evolution is a function of the rate of Combustion and of the surface under Ignition. We may therefore say, that the pressure is a function of the rate of Combustion and of the surface jointly. But the rate of Combustion is itself a function of the pressure, so that in fact, the pressure is a function of the ignited surface, and making the time the independent variable, the quantity of gas evolved and the pressure are functions of the form of grain on which the variation of the surface depends.

52. There is, however, in the case of a gun a further complication. The space is also variable, and this of course affects the pressure behind the projectile.

53. With a very quick burning powder the projectile has moved very little at the time when the charge is entirely consumed, while with a slow evolution of gas, it has moved a considerable distance, thus greatly increasing the space and decreasing the maximum pressure.

There is a contest going on between the rapidly increasing space and the increasing evolution of gas. The former increases in an increasing ratio with the time, whilst the latter, though it increases, does so in a decreasing ratio with the time, because the ignited surface in every ordinary form of grain decreases as the burning goes on. Then again, the evolution of gas increases with the pressure, and is thus an

increasing ratio up to the point of maximum pressure, and a decreasing ratio as regards the time, after.

54. If, at the time of the maximum pressure, the whole charge is consumed, the subsequent pressures behind the projectile will be represented by an adiabatic curve, and the work done on the projectile easily ascertainable, but as regards the work previously done, it is obvious, from what is said above, that the problem is an exceedingly complicated one.

55. If the charge be not entirely consumed at the time of the maximum pressure, one of two things may take place: either the maximum pressure may be sustained for a period, owing to the effect of the increasing space being exactly compensated by the increasing evolution of gas (which, however, is not possible with any ordinary form of grain), or the pressure may go on falling, but less rapidly than the adiabatic curve, owing to the continued evolution of heat and gas by the remaining unconsumed powder.

In this case it is evident, that at the time when the whole of the charge is consumed, the pressure will be the same as in the case of the other powder and the subsequent pressures follow the same law. Up to this point the pressures must always be less, and therefore the whole work done on the projectile must be less with a slow than with a quick powder, and to obtain an equal ballistic effect larger charges must be used.

This is confirmed in practice and it is also in conformity with the thermodynamic law, "that any thermal machine which works between given limits of temperature gives the maximum effect when all the heat is received at the highest temperature and rejected at the lowest."

Form of Grain.

56. The influence of the form of grain upon the pressure and evolution of gas is very great, and although the relation between them and the distance moved by the projectile is

very complicated, an approximate idea of the influence of form may be obtained by considering the evolution of gas in a close vessel as a function of the time.

57. Let it be assumed, that the composition, density, and degree of humidity are the same, the only difference being in the *form* of the grain. Let three forms of grain be considered, the spherical, cubical, and prismatic or cylindric, with a central hole. Further, that the weight of the grains is the same, and that the whole surface is simultaneously ignited.

58. Since the space is constant, the pressure is a function of the time, and may be considered as equal to some unknown power of it.

The rate of burning is, as stated before, proportional to some power of the pressure. We may therefore assume the velocity of burning proportional to some function of the time, or

$$v = n\,(1 + t^m),$$

where n is the rate of burning in free air, t the time, m an unknown power of it.

On this assumption the volume of powder consumed may be found as a function of the time, as follows:—

Spherical Grain.

59. Let d = diameter of the grain;
s = distance from the original surface burnt at the time t;
S = actual surface under ignition at t;

then

$$S = \pi\,(d - 2\,s)^2 = \pi\,(d^2 - 4\,d\,s + 4\,s^2). \qquad (1)$$

The velocity of burning at $s = \dfrac{d\,s}{d\,t}$,

which by assumption

$$= n\,(1 + t^m);$$

therefore

$$\frac{d\,s}{d\,t} = n\,(1 + t^m),$$

and integrating and observing that when $t = 0$ $s = 0$,

$$S \quad \cancel{s} = n\left(t + \frac{t^{m+1}}{m+1}\right).$$

$$S^2 \quad \cancel{s^2} = \frac{n^2}{(m+1)^2}\left\{(m+1)^2 t^2 + 2(m+1)t^{m+2} + t^{2(m+1)}\right\}.$$

Substituting this in (1),

$$S = \pi\left\{d^2 - \frac{4dn}{m+1}\left((m+1)t + t^{m+1}\right)\right.$$
$$\left. + \frac{4n^2}{(m+1)^2}\left((m+1)^2 t^2 + 2(m+1)t^{m+2} + t^{2(m+1)}\right)\right\}$$

and as the thickness burnt in dt is $n(1 + t^m)\,dt$, making V the volume burnt,

$$dV = \pi n\left\{d^2(1 + t^m)\,dt - \frac{4dn}{m+1}\left((m+1)(1 + t^m)\,t\,dt\right.\right.$$
$$\left. + t^{m+1}(1 + t^m)\,dt\right) + \frac{4n^2}{(m+1)^2}\left((m+1)^2(1 + t^m)\,t^2\,dt\right.$$
$$\left.\left. + 2(m+1)(1 + t^m)\,t^{m+2}\,dt + t^{2(m+1)}(1 + t^m)\,dt\right)\right\}.$$

Integrating and observing that when $t = 0$ $V = 0$, we get finally

$$V = \pi n d^2\left(t + \frac{t^{m+1}}{m+1}\right) - \frac{4\pi d n^2}{m+1}\left(\frac{m+1}{2}t^2 + t^{m+2} + \frac{t^{2(m+1)}}{2(m+1)}\right)$$
$$+ 4\pi n^3\left(\frac{t^3}{3} + \frac{t^{m+3}}{m+1} + \frac{t^{2m+3}}{(m+1)^2} + \frac{t^{3(m+1)}}{3(m+1)^3}\right).$$

Cubical Grain.

60. In this case let d be the side of the cube, s as before, then $d - 2s$ is the length of the side at t and $6\,(d - 2s)^2$ the surface under ignition. This only differs from the case of the spherical grain by substituting 6 for π, and making d_1 the side of the cube instead of the diameter of the sphere.

Consequently the formula for the spherical grain was

$$V = 6nd_1^2\left(t + \frac{t^{m+1}}{m+1}\right) - 24\,d_1 n^2\left(\frac{t^2}{2} + \frac{t^{m+1}}{m+1} + \frac{t^{2(m+1)}}{2(m+1)^2}\right)$$
$$+ 24\,n^3\left(\frac{t^3}{3} + \frac{t^{m+3}}{m+1} + \frac{t^{2m+3}}{(m+1)^2} + \frac{t^{3(m+1)}}{3(m+1)^3}\right).$$

61. This formula may be written for both grains,

$$V = A\left(t + \frac{t^{m+1}}{m+1}\right) - B\left(\frac{t^2}{2} + \frac{t^{m+2}}{m+1} + \frac{t^{2(m+1)}}{2(m+1)^2}\right)$$
$$+ C\left(\frac{t^3}{3} + \frac{t^{m+3}}{m+1} + \frac{t^{2m+3}}{(m+1)^2} + \frac{t^{3(m+1)}}{3(m+1)^3}\right).$$

		For Spherical Grain.	For Cubical Grain.
Where	$A =$	$\pi\, n\, d^2$	$6\,n\,d_1$
	$B =$	$4\,\pi\, d\, n^2$	$24\,d_1\,n^2$
	$C =$	$4\,\pi\, n^3$	$24\,n^3$

Prismatic Grain.

62. Let ρ be the radius of the central hole.
R the distance from centre to flat side.
T the thickness of the prism.
S the length of the flat side of the hexagon.

Then proceeding as before, we find

$$V_1 = A_1\left(t + \frac{t^{m+1}}{m+1}\right) + \frac{B}{m+1}\left(\frac{t^{2(m+1)}}{2(m+1)} + t^{m+2} + \frac{m+1}{2}\,t^2\right)$$
$$+ \frac{C}{(m+1)^2}\left(\frac{t^{3(m+1)}}{3(m+1)} + t^{2m+3} + \frac{(m+1)^2 + 2(m+1)}{m+3}\,t^{m+3}\right.$$
$$\left. + \frac{(m+1)^2}{3}\,t^3\right);$$

where

$$A_1 = n\left\{\frac{6\,S}{R}(T\,R + R^2) + 2\,\pi\,\rho\,(T - \rho)\right) = n\left(6\,S\,(T + R)\right.$$
$$\left. + 2\,\pi\,\rho\,(T - \rho)\right\}.$$

$$B_1 = n^2 \left\{ 2\pi (T - 4\rho) - \frac{6S}{R}(4R + T) \right\}.$$

$$C_1 = n^3 \left\{ \frac{18S}{R} - 6\pi \right\}.$$

63. In the forms of grain above dealt with, the surface of Ignition decreases as the Combustion proceeds. In the prismatic form, however, with the central hole, one portion of the surface, viz. the ends and outside, decreases, while the central hole increases.

64. There is, however, another form patented by Mr. Quick, in which he has sought to give a greater proportion to the increasing surface by making the powder into discs of the same diameter as the chamber of the gun and piercing it with a number of cylindrical holes, and these discs are so connected together that when a number of them are made up into a cartridge, the cylindric holes will be continuous throughout the whole length. The flat discs are moreover dished out on one of their surfaces, so as to make these surfaces also surfaces of Combustion.

If then R = radius of disc;
ρ = radius of the holes;
ν = number of the holes;
T = the thickness of disc;

the increasing surface is

$$2\pi\nu\rho T,$$

the decreasing surface,

$$2\pi R T + 2\pi (R^2 - \nu\rho^2),$$

and if x be the distance burnt at the end of t, the increasing surface is

$$2\pi\nu(\rho + x)(T - 2x),$$

the decreasing surface,

$$2\pi(R - x)(T - 2x) + 2\pi(R - x)^2 - [\nu(\rho + x)^2];$$

or the total surface under ignition at t

$$= 2\pi\{\nu(\rho + x)(T - 2x) + (R - x)(T - 2x) + (R - x)^2 - \nu(\rho + x^2)\},$$

and as was shown before,

$$x = n\left(t + \frac{t^{m+1}}{m+1}\right);$$

substituting which, and multiplying by dt, and integrating we get

$$V = A\left(t + \frac{t^{m+1}}{m+1}\right) + \frac{B}{m+1}\left(\frac{m+1}{2}t^2 + t^{m+2} + \frac{t^{2(m+1)}}{2(m+1)}\right) + \frac{C}{(m+1)^2}\left(\frac{(m+1)^2}{3}t^3 + \frac{2(m+1)+(m+1)^2}{m+3}t^{m+3} + \frac{(2m+3)t^{2m+3}}{2m+3} + \frac{t^{3m+3}}{3m+3}\right).$$

where

$$A = 2\pi n\{R(T + R) + \nu\rho(T - \rho)\}.$$
$$B = 2\pi n^2\{T(\nu - 1) - 4(R - \nu\rho)\}.$$
$$C = 2\pi n^3\{3(1 - \nu)\}.$$

65. The rate of burning has in all these cases been assumed to be a function of the time represented by $v = n(1 + t^m)$ where n is the rate of burning in free air, and the unit of time is taken as the ten-thousandth of a second, therefore

$$n = \cdot 0004 \text{ in.}$$

To illustrate the formula I will assume three values of m.

$m = -\infty$ giving $v = n$ or uniform rate.
$m = \frac{1}{2}$ giving $v = n(1 + t^{\frac{1}{2}})$ increasing as $t^{\frac{1}{2}}$.
$m = 1$ $v = n(1 + t)$ increasing as t.

66. The formula then becomes:—

For spherical and cubical grain,—

when $m = -\infty$ $V = At - \frac{B}{2}t^2 + \frac{C}{3}t^3;$

when $m = \frac{1}{2}$ $V = A\left(t + \frac{t^{1\cdot 5}}{1\cdot 5}\right) - B\left(\frac{t^2}{2} + \frac{t^{2\cdot 5}}{1\cdot 5} + \frac{t^3}{4\cdot 5}\right) + C\left(\frac{t^3}{3} + \frac{t^{3\cdot 5}}{1\cdot 5} + \frac{t^4}{2\cdot 25} + \frac{t^{4\cdot 5}}{10\cdot 125}\right);$

when $m = 1$, $$V = A\left(t + \frac{t^2}{2}\right) - B\left(\frac{t^2}{2} + \frac{t^3}{2} + \frac{t^4}{8}\right) + C\left(\frac{t^3}{3} + \frac{t^4}{2} + \frac{t^5}{4} + \frac{t^6}{24}\right).$$

For prismatic grain,—

when $m = -\infty$, $V = At + \frac{B}{2}t^2 + \frac{C}{3}t^3$;

when $m = \frac{1}{2}$, $$V = A\left(t + \frac{t^{1\cdot5}}{1\cdot5}\right) + B\left(\frac{t^2}{2} + \frac{t^{2\cdot5}}{1\cdot5} + \frac{t^3}{4\cdot5}\right) + C\left(\frac{t^3}{3} + \frac{t^{3\cdot5}}{1\cdot5} + \frac{t^4}{2\cdot25} + \frac{t^{4\cdot5}}{10\cdot125}\right);$$

when $m = 1$, $$V = A\left(t + \frac{t^2}{2}\right) + B\left(\frac{t^2}{2} + \frac{t^3}{2} + \frac{t^4}{8}\right) + C\left(\frac{t^3}{3} + \frac{t^4}{2} + \frac{t^5}{4} + \frac{t^5}{24}\right).$$

For disc powder,—

when $m = -\infty$, $V = At + \frac{B}{2}t^2 + \frac{C}{3}t^3$;

when $m = \frac{1}{2}$, $$V = A\left(t + \frac{t^{1\cdot5}}{1\cdot5}\right) + B\left(\frac{t^2}{2} + \frac{t^{2\cdot5}}{1\cdot5} + \frac{t^3}{4\cdot5}\right) + C\left(\frac{t^3}{3} + \frac{t^{3\cdot5}}{1\cdot5} + \frac{t^4}{2\cdot25} + \frac{t^{4\cdot5}}{10\cdot125}\right);$$

when $m = 1$, $$V = A\left(t + \frac{t^2}{2}\right) + B\left(\frac{t^2}{2} + \frac{t^3}{3} + \frac{t^4}{8}\right) + C\left(\frac{t^3}{3} + \frac{t^4}{4} + \frac{t^5}{4} + \frac{t^6}{24}\right).$$

It will be seen by comparing these expressions that they only differ in the coefficients, A, B, and C, and that the terms involving the power of t for any given value of m, are identical, showing that the coefficients depend upon the form of grain.

67. The following table gives the value of these coefficients, for the four forms of grain above considered.

	Spherical.	Cubical.	Prismatic with one hole.	Disc with ν holes.
A	$4\pi n R^2$	$6 n S^2$	$n\{6S(T+R)+2\pi\rho(T-\rho)\}$	$2\pi n\{R(T+R)+\nu\rho(T-\rho)$
B	$8\pi n^2 R$	$24 n^2 S$	$n^2\left\{2\pi(T-4\rho)-\frac{6S}{R}(T+4R)\right\}$	$2\pi n^2\{T(\nu-1)-4(R-\nu\rho)\}$
C	$4\pi n^3$	$24 n^3$	$n^3\left\{\frac{18S}{R}-6\pi\right\}$	$2\pi n^3\{3(1-\nu)\}$

where $n = \cdot 0004$;

R = radius of sphere, or disc, or in the case of prismatic, the radius of the inscribed circle;

ρ = radius of perforating holes;

S = side of cube or hexagon in prismatic powder;

T = thickness of prismatic grain or disc;

ν = number of holes in disc.

68. The relative evolution of gas, as influenced by the form of grain, may be shown graphically by diagrams constructed from the above formula.

Let it be assumed, that in each of the three first forms of grain, the grain is of the same weight, and that each contains 1·572 cubic inches of powder, corresponding to the following dimensions, say,

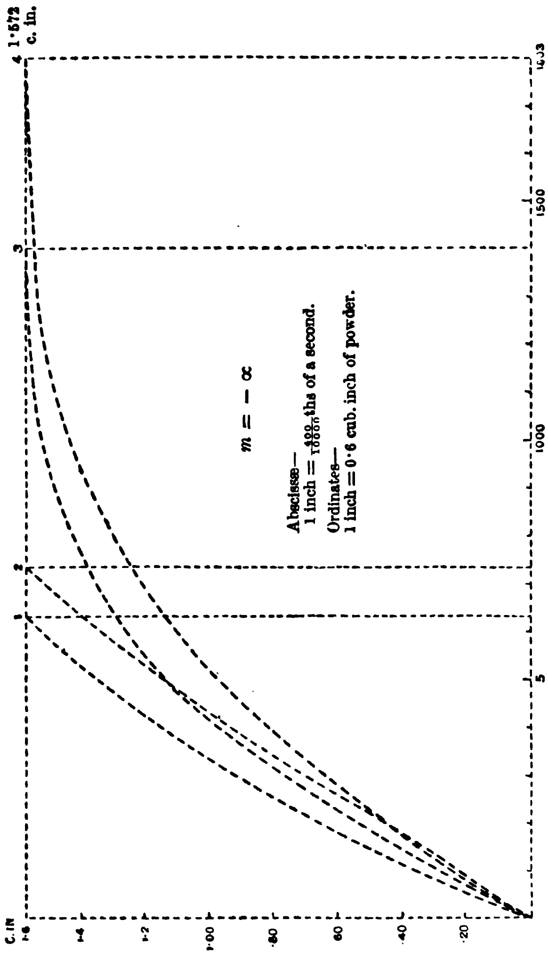
			inches.
Spherical	..	diameter	1·4426
Cubical ..	..	length of side ..	1·1628
Prismatic	$2R$ =	across flats	1·4000
	2ρ =	diameter of holes ..	·4000
	S =	side of hexagon ..	·8083
	T =	thickness	1·0000
Disc ..	R =	outer radius	6·0000
	ρ =	radius of holes ..	·1000
	ν =	number of holes ..	19 holes
	T =	thickness	1·0000

and let the disc powder be of the following dimensions :—

	inches.
Outer radius of disc	3
Radius of holes	0·1
Thickness of disc	1·0
Number of holes	19

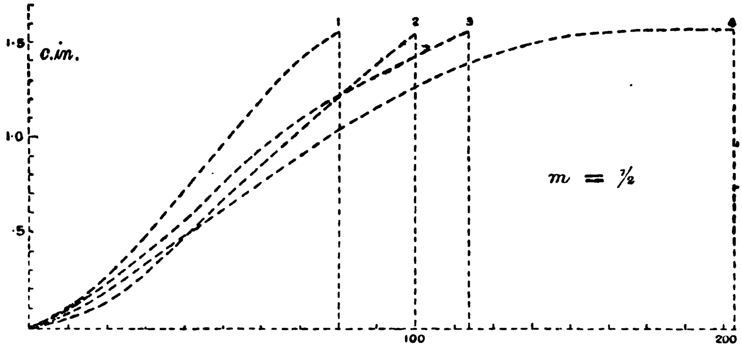
As the volume of this disc is fourteen times greater than that of one of the above grains, for the sake of comparison, the volume given by the formula must be divided by fourteen, as has here been done.

FIG. 1.



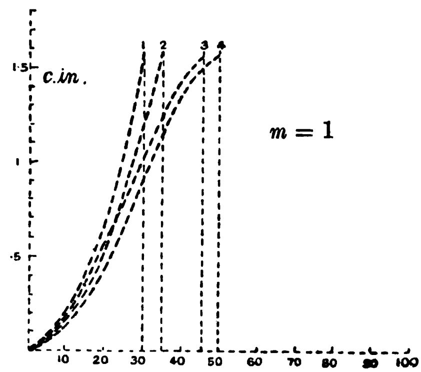
69. Fig. 1 represents the volume of powder converted into gas as a function of the time, on the assumption of $m = -\infty$, or a uniform rate of burning.

FIG. 2.



Figs. 2 and 3 represent the same on the hypotheses, that $m = \frac{1}{2}$, and $m = 1$ respectively.*

FIG. 3.



Abscissæ 1 inch = $\frac{50}{10000}$ths of a second.
Ordinates 1 inch = 1 cub. inch powder.

The following tables show, in each case, the time of consuming 1·572 cubic inch of powder, in ten-thousandths of a second.

* In these diagrams the curve 1 represents Prismatic; 2 represents Disc; 3 represents Cubical Grain; 4 represents Spherical Grain.

D

Value of m.	Time of consumption of 1·572 cubic inch.		
	$-\infty$	$\frac{1}{2}$	1
Spherical	1803	182	50
Cubical	1453	113	45
Disc	730	101	35
Prismatic	625	79	29

70. From this it appears, that whatever be the form of grain, the prismatic is that which burns quickest, and consequently, as far as mere form is concerned, it ought to give the highest pressure.

But practically this is not the case, partly because prismatic powder is generally of a higher density, but chiefly because the ignition is slow at first, owing to the high glazing and the small surface.

The cartridges are so built up, that the grains fit into each other, so that practically the initial surface of ignition is almost limited to that of the central holes. The first evolution of gas is therefore small, and the initial pressure rises slowly, and as soon as it is sufficient to overcome the friction of the projectile and the resistance of the base ring, the projectile moves away, and by the time the evolution of gas becomes rapid, the projectile is already some distance along the bore, and the increased space keeps down the pressure.

71. It will be seen from the diagrams, that the evolution of gas is much more uniform with the disc and prismatic powders than with spherical and cubical. This is no doubt an advantage, but it is inconsistent with what is often asserted, viz. that with prismatic powder, the combustion continues a long way down the bore. The real advantage of this form is that whilst the evolution of powder gas is small at first, owing to the limited surface of combustion, it increases very rapidly as the combustion goes on, and thus, though the maximum pressure

is reduced, the mean pressure during the process of combustion is greater in the early stage of the projectile's motion.

The maximum pressure is a direct function of the evolution of gas and an inverse function of the distance moved by the projectile, and the former is directly as the surface and the velocity of combustion. The surface of a charge increases rapidly as the size of the grain decreases, and the total time of combustion of the charge (supposing perfect ignition) is a function of the least dimension of the grain, so that the evolution of gas is much more rapid with small-grained powders, both from the extended surface and from the reduced depth to be burnt, than with large-grained powders of the same form, and the maximum pressure is still more kept down in the case of disc and prismatic powders for the reasons above stated.

72. With regard to the disc powder, it may be observed that the variation of the surface of combustion may be regulated by the thickness of the disc. For, if x be the depth burnt at any time, the surface of combustion at that time is

$$S = 2\pi \{R(T+R) + \nu\rho(T-\rho)\} + \{T(\nu-1) - 4(R+\nu\rho)\}x + 3(1-\nu)x^2 = 2\pi(\alpha + \beta x + \gamma x^2);$$

but

$$\frac{dS}{dx} = 2\pi\{\beta + 2\gamma x^2),$$

and in the case of a maximum this is equal to zero, when

$$x = -\frac{\beta}{2\gamma} = \frac{4(R+\nu\rho) - T(\nu-1)}{6(1-\nu)} = \frac{T(\nu-1) - 4(R+\nu\rho}{6(\nu-1)}.$$

Now at the beginning of combustion $x = 0$, therefore if the maximum value of the surface is at the beginning, $x = 0$ and $T = \dfrac{4(R+\nu\rho)}{\nu-1}$.

74. In order that the surface may be an increasing surface during the whole time of combustion, we must make x equal to

one-half the distance between the holes, and calling this Δ we must make

$$\frac{\Delta}{2} = \frac{T(\nu - 1) - 4(R + \nu\rho)}{6(\nu - 1)}.$$

In the case of the disc powder above considered, $\Delta = 0 \cdot 8$, $\nu = 19$, $R = 3$, $\rho = 0 \cdot 1$, making use of which values we find $T = 3 \cdot 49$ inches, the required thickness.

75. Thus, theoretically, it is always possible to regulate the thickness of the disc so as to have an always increasing surface of combustion, but probably this will not be possible in practice, on account of the difficulty of obtaining uniformity of density in thick discs.

It is, however, proposed by Mr. Quick to unite a number of these discs by means of a very inflammable cement, and I believe he has had considerable success with this method.

If this can be accomplished, I have no doubt that cartridges made up of discs thus united, will give excellent results, combining a very rapid subsequent evolution of gas with a comparatively low maximum initial pressure, and such cartridges made up with an envelope of fusible metal will be found very convenient, especially for quick-firing guns, the cartridge case disappearing with the products of combustion and requiring no extraction.

Products of Combustion.

76. In (§ 28) it was stated that about 43 per cent. of the weight of the products consist of permanent gases, and 57 per cent. of compounds which solidify at ordinary temperatures. As the ballistic effect of gunpowder is simply a conversion of a portion of the heat evolved into mechanical force, by means of the expansive action of the permanent gases, a gun is just as much a thermal machine as is a steam or an air engine.

If then the initial and final temperatures of the gases could

be ascertained, the fall of temperature, subject to certain deductions, would give the energy of the projectile. These deductions are, the energy of the products of combustion, the energy of recoil of the gun and carriage, the force required to give rotation in rifled guns, and the friction of the projectile and the escaping gases.

77. This method of treatment of the ballistic problem is due to Count St. Robert who devotes some space to it in his 'Traité de Thermodynamique,' Turin, 1870, chapter iii., and an attempt to further develope it formed the subject of a paper presented by myself to the Institution of Civil Engineers in 1884, and published in the Minutes of Proceedings, vol. xxx., part ii. It is further dealt with in a subsequent chapter of the present work.

78. The volume of permanent gases, and the units of heat evolved from the combustion of a given weight of powder are fundamental data in ballistic problems, and the following table gives these for several descriptions of powder, as determined by Messrs. Noble and Abel.

79. Table of cubic centimetres of gas and units of heat evolved per gramme of powder:—

Description of Powder.	*a.* Gramme units evolved from 1 gramme of powder.	*b.* Cubic centimetres of gas evolved from 1 gramme of powder.	Products of *a* and *b*.
Cocoa—Brown prismatic	837·0	198·0	165,720
Spanish pellet	767·3	234·2	179,680
Curtis and Harvey No. 6	764·4	241·0	184,220
Waltham Abbey, F.G...	738·3	263·1	194,250
" R.L.G.	725·7	274·2	198,980
" Pebble	721·4	278·3	200,770
Mining powder	516·8	360·3	186,200

80. It will be at once observed, that those powders which evolve the most gas give out the least heat, which is no

doubt due to the disappearance of sensible heat, in giving the gaseous form to the products.

As the pressure when confined in a close vessel is a function of the temperature and of the volume jointly, the figures given in the last column represent to a certain extent the relative pressures from the above powders. Thus the Waltham Abbey powders are pretty nearly equal, whilst the mining powder, Spanish, and Curtis and Harvey No. 6 are somewhat less, and the cocoa the least of all. The Curtis and Harvey No. 6 and the mining powder are nearly the same, although the proportions of gas and units of heat differ by nearly 50 per cent.

81. The proportions by weight of the gaseous and solid or liquid portions of the products are shown in the following table, also due to Messrs. Noble and Abel.

Description of Powder.	Water pre-existent.	Gaseous.	Solid or liquid.
Cocoa—Brown prismatic	·0333	·4175	·5825
Spanish pellet	·0065	·3808	·6127
Curtis and Harvey No. 6	·0117	·4109	·5774
Waltham Abbey, F.G...	·0148	·4282	·5569
" R.L.G.	·0106	·4298	·5591
" Pebble	·0095	·4409	·5496
Mining powder	·0161	·5135	·4201

Temperature of Combustion.

83. If the specific heats of the various products of combustion were known, it would be easy to obtain the temperature of combustion, but unfortunately this specific heat, though ascertainable at ordinary temperatures, increases with the temperature according to an unknown law. Consequently the temperatures as deduced from the specific heats at ordinary temperatures can only be taken as superior limits.

They will, however, approximately represent the relative temperatures of combustion, which may be found as follows :—

w Let W be the weight of powder;

H the units of heat evolved per unit of weight;

s the mean specific heat of the products;

T the temperature of the products;

then

$$H = w s T, \quad \text{or} \quad T = \frac{H}{w s}.$$

And if w be taken = 1 gramme

$$T = \frac{H}{s}.$$

84. The mean specific heat at constant volume, of the products of combustion at ordinary temperatures have been determined by Messrs. Noble and Abel, and the following table shows the corresponding temperatures, the units of heat evolved being taken from the table (§ 79).

85.

Description of powder.	Gramme units of heat evolved per gramme.	Mean specific heat.	Temperature Centigrade.
Cocoa—Brown prismatic	837·0	·20000	4185°
Spanish pellet	767·3	·18773	4087
Curtis and Harvey No. 6	764·4	·18720	4084
Waltham Abbey, F.G...	738·3	·18946	3898
" R.L.G.	725·7	·18704	3881
" Pebble	721·4	·18503	3900
Mining powder	516·8	·17846	2895

86. There can be little doubt that these are above the actual temperatures in a gun, but they probably represent the relative temperatures. There is, however, another method of calculation which probably gives pretty nearly the actual temperatures. It is obtained from a formula given hereafter,

$$T_0 = \frac{273 f}{p_0 v_0}$$

when T_0 is the absolute temperature taken from 273° below zero of the Centigrade scale.

f is the pressure of the gases arising from 1 kilog. of powder occupying at the temperature of combustion, 1 centimetre cube of space.

p = atmospheric pressure or 1·033 kilog. per square centimetre.

v_0 = volume of gases from unit of weight.

Now for pebble powder and R.L.G. the value of f is about 2615 kilog. per centimetre, and $v_0 = 276$.

Therefore,

$$T_0 = \frac{273 \times 2615}{1\cdot033 \times 276} = 2504^\circ \text{ absolute,}$$

or by the Centigrade thermometer = 2504 − 273 = 2231° C. which is probably approximately true.

87. If this be so the mean specific heats may be determined by dividing the units of heat evolved by unit of weight, by the temperature just found. This gives for pebble powder

$$\frac{721\cdot4}{2231} = 0\cdot322,$$

which is about 75 per cent. higher than the specific heat made use of in the table (§ 85).

If the specific heats used in that table be increased in the same proportion, the resulting temperatures will be

Description of Powder.	Temperature.
Cocoa—Brown prismatic	2390° C.
Spanish pellet	2335
Curtis and Harvey No. 6 ..	2334
Waltham Abbey, F.G.	2225
„ R.L.G. ..	2218
„ pebble	2230
Mining powder	1654

The following table is given by Captain Roulin of the French Artillery, and is derived from the experiments of Messrs. Noble and Abel, and Messrs. Roux and Sarrau.

	Description of Powder.	Gramme units of heat evolved per gramme of powder.	Volume of gas evolved, cubic centimetres per gramme.	Temperature of combustion.	Force in kilogrammes per square centimetre.
English.	Pebble	721·4	276	2241° C.	2615
	R.L.G.	725·7	271	2250	2560
	F.G.	738·3	259	2280	2520
	Spanish pellet	767·2	233	2350	2300
	Curtis and Harvey No. 6	744·4	238	2340	2340
	Mining powder	516·8	355	1610	2510
French.	Fine poudre de chasse	810	234	2500	2450
	Ordinary cannon powder	756	261	2340	2350
	Musket powder A ..	732	280	2260	2680
	Mining powder	574	316	1770	2340

88. The high temperature of combustion of cocoa powder, accompanied as it is by an increased weight of the solid or liquid residue, and also by the increased charges required to keep up the ballistic effect, may probably have a good deal to do with the rapid erosion of the bore in modern artillery practice.

Strength of Powder.

89. By the term "strength" is denoted the pressure per unit of surface which the products of combustion exert on the sides of a close vessel which is filled with the powder at gravimetric density = 1 or 27·70 inches per pound of powder.

The absolute strength under these circumstances has not yet been ascertained, because in all experiments made to that end, a certain amount of the heat evolved passes into the substance of the containing vessel, and consequently is lost as regards ballistic power. The proportion thus lost cannot be a constant ratio, because whilst the heat actually evolved is directly as the weight, or as the cube of the lineal dimensions of the vessel, the surface exposed varies as the lineal dimensions, as regards the sides, and as the square of the lineal dimensions as regards the ends. Consequently, the loss of heat is greater in proportion in a small vessel than in a large one.

90. Much uncertainty exists with regard to the actual amount of cooling in a gun due to the transmission of heat to the metal.

Count St. Robert, by his experiments on small arms, concluded that about one-third of the whole heat evolved was thus absorbed, or about 250 units per kilo. of powder.

Messrs Noble and Abel who experimented with a 12-pounder, estimated the loss at about 100 units, and in a 10-inch gun at not more than 25 units per kilog. of powder, being about 14 per cent. and 3½ per cent. respectively.

91. On this subject M. Sarrau has made some interesting remarks which may be stated as follows.

92. Let w be the weight of powder burnt in a close vessel.

T_0 the initial temperature of combustion.

T the temperature of the products at any time t.

σ the surface of the vessel.

v the rate of flow of heat in units per unit of surface and unit of time.

Unities: decimetre, kilogramme, second, French unit of heat, degree Centigrade.

93. The combustion is not instantaneous but progressive, and at the end of any time t the quantity burnt will be some function of t. Let this be denoted by F (t).

At the time t the heat absorbed may be represented by

$$\sigma \int_0^t v\,dt. \tag{1}$$

On the other hand, the heat lost by the products in falling from T_0 to T_1 is

$$c\,F(t)(T_0 - T), \tag{2}$$

c being the specific heat under constant volume, therefore

$$c\,F(t)(T_0 - T) = \sigma \int_0^t v\,dt. \tag{3}$$

94. Now v is a function of the temperatures of the vessel and of the products of combustion, and depends also on the emissive power of the products, and on the absorptive power of the vessel.

According to Dulong and Petit as quoted by M. Sarrau,

$$v = \mathrm{H}\, e\, e' \left(a^{\mathrm{T}} - a^{\mathrm{T}_1}\right); \qquad (4)$$

where T and T_1 are the absolute temperatures of the products of combustion and the walls of the vessel;

e, the emissive power of the products;

e', the absorptive power of the vessel;

a, a constant $= 1 \cdot 0077$;

H, a constant;

95. Now T_1 is always small relative to T, so that a^{T_1} may be neglected and we get

$$v = \mathrm{H}\, e\, e'\, a^{\mathrm{T}}. \qquad (5)$$

Therefore

$$c\, \mathrm{F}\,(t)\,(\mathrm{T}_0 - \mathrm{T}_1) = \mathrm{H}\, e\, e'\, \sigma \int_0^t a^{\mathrm{T}}\, d\, t \qquad (6)$$

and writing z for $\mathrm{T}_0 - \mathrm{T}$ the fall of temperature

$$z\, \mathrm{F}\,(t) = \frac{\mathrm{H}\, e\, e'\, \sigma}{c} \int_0^t a^{\mathrm{T}}\, d\, t. \qquad (7)$$

But

$$\mathrm{T} = \mathrm{T}_0 - z;$$

Therefore

$$a^{\mathrm{T}} = a^{\mathrm{T}_0 - z} = \frac{a^{\mathrm{T}_0}}{a^z}$$

and (7) becomes

$$z\, \mathrm{F}\,(t) = \frac{\mathrm{H}\, e\, e'\, \sigma\, a^{\mathrm{T}_0}}{c} \int_0^t a^{-z}\, d\, t. \qquad (8)$$

96. Whatever be the law of combustion, F (t) may generally be developed according to ascending powers of t in the form

$$\mathrm{F}\,(t) = w\, a\, t\,(1 + \lambda\, t + \mu\, t^2 + \&c.).$$

and developing z in the same form, or making

$$z = z_0 + z_1 t + z_2 t^2 \text{ \&c. \&c.}$$

and substituting in (6) the coefficients may be determined.

97. At present, we will suppose that the law of burning is uniform, then if τ be the time of total combustion of a grain and of the charge

$$F(t) = w \frac{t}{\tau} \tag{9}$$

making

$$\frac{H e e' \sigma a^{T_0}}{c} = \kappa$$

(8) becomes

$$z F(t) = \kappa \int_0^t a^{-z} dt \tag{10}$$

or

$$z w \frac{t}{\tau} = \kappa \int_0^t a^{-z} dt \tag{11}$$

which is satisfied by attributing to z a value independent of t such that $\kappa a^{-z} = w \frac{z}{\tau}$,

or

$$z a^z = \kappa \frac{\tau}{w}. \tag{12}$$

98. The hypothesis of a uniform rate of combustion is as we know incorrect, because in a close vessel the pressure increases as the time, and the rate of burning increases as the pressure; on the other hand, the surface of the grains generally decreases as the time increases, so that there may possibly be no great error in the hypothesis of uniformity.

99. The equation (12) is easily solved when κ is known, for taking the logarithms

$$\left.\begin{aligned} &\log z + z \log a = \log \kappa + \log \tau - \log w, \\ \text{or} \quad &\log z + z \log a = \log \left(\frac{H e e'}{c} a^{T_0} \tau\right) + \log \frac{\sigma}{w} \end{aligned}\right\} \tag{13}$$

But

$$\kappa = \frac{\mathrm{H}\, e\, e'\, \sigma\, a^{\mathrm{T}_0}}{c},$$

and

$$\log \kappa = \log \mathrm{H} + \log e + \log e' + \log \sigma + \mathrm{T}_0 \log a - \log c. \quad (14)$$

In dealing with high temperatures the term $\mathrm{T}_0 \log a$ is very large when compared with the other terms, consequently a considerable error in the estimation of e or e' will not much affect the results.

100. Now the value of e the emissive power of the products is absolutely unknown, but as these products consist to a large extent of solid or liquid matter in a highly incandescent state, the emissive power is no doubt very great, and there will probably be no great error in taking $e = 1$.

The value of e' depends on the nature and state of the material of the vessel. M. Sarrau gives to e' the value of 0·15, and to c the specific heat the value 0·185. He takes $\mathrm{T}_0 = 4080°$ the theoretic value obtained by $\mathrm{T}_0 = 274 + \frac{\mathrm{Q}}{c}$ when Q = 705 units, and H = ·000237,* making use of which values he finds

* This value of H is deduced by M. Sarrau from an experiment made by Dulong and Petit ('Journal de l'École Polytechnique,' cahier xviii. p. 257), on the cooling of the bulb of a mercurial thermometer covered with lampblack in a medium at 0° C., or absolute temperature T = 272°, as follows:—

Let h be the quantity of heat passing in unit of time and surface $= \mathrm{H}\, e\, e'\, (a^{\mathrm{T}} - a^{\mathrm{T}_1})$.

Then in δt, the heat passing $= h\, \sigma\, \delta t$, and if w be the weight of the cooling body, c its specific heat, then the decrease of temperature $\delta \mathrm{T} = \frac{h\, \sigma}{w\, c}\, \delta t$, and the thermometric velocity of cooling $h' = \frac{h\, \sigma}{w\, c}$.

Substituting for h its value $\mathrm{H}\, e\, e'\, (a^{\mathrm{T}} - a^{\mathrm{T}_1})$, we get

$$h' = \frac{\mathrm{H}\, e\, e'\, \sigma}{w\, c} (a^{\mathrm{T}} - a^{\mathrm{T}_1}).$$

Let

ρ = radius of bulb in decimetres, Δ = density of mercury,

then

$$\sigma = 4\pi\rho^2 \text{ and } w = \frac{4}{3}\pi\rho^3\Delta, \text{ therefore } \frac{\sigma}{w} = \frac{3}{\rho\,\Delta}.$$

$$\begin{aligned}\log \kappa &= \log \mathrm{H} + \log e + \log \epsilon' + \log \sigma + \mathrm{T}_0 \log a - \log c \\ &= \log \cdot 000237 + \log 1 + \log \cdot 15 + 4080 \log 1 \cdot 0077 \\ &\quad - \log \cdot 185 + \log \sigma \\ &= 9 \cdot 874 + \log \sigma,\end{aligned}$$

and (13) becomes

$$\log z + z \log a = 9 \cdot 874 + \log \tau + \log \frac{\sigma}{w}. \qquad (15)$$

101. If then the vessel be a cube of 1 decimetre, $\sigma = 6$, and giving various values to T and w the corresponding values of z may be calculated.

Assuming T and w as follows

$$\begin{array}{llll} \mathrm{T} = 0 \cdot 100 & 0 \cdot 01 & 0 \cdot 001 & \text{seconds,} \\ w = 0 \cdot 1 & 0 \cdot 5 & 1 \cdot 00 & \text{kilogrammes,} \end{array}$$

the value of z, the fall of temperature, will be found as in the following table.

T, seconds.	w in kilogrammes.		
	0·1	0·5	1·0
	z	z	z
	°	°	°
·001	1630	1440	1360
·020	1910	1720	1630
·100	2190	2000	1910

and if θ = difference of temperature or $\theta = \mathrm{T} - \mathrm{T}_1$

$$h' = \mathrm{H} \frac{3\, e\, e'\, a^{\mathrm{T}_1}}{c\, \rho\, \Delta} (a^\theta - 1).$$

Dulong took $e = 0 \cdot 8$, e' = unity, $\mathrm{T}_1 = 273$, $c = \cdot 0333$, $\Delta = 13 \cdot 596$, $\rho = 0 \cdot 3$, and the velocity of cooling was found to be represented by $h' = \cdot 03395 (a^\theta - 1)$. Equating these two

$$\cdot 03395 = \frac{\mathrm{H}\, 3\, e\, e'\, a^{\mathrm{T}_1}}{c\, \rho\, \Delta}$$

$$\therefore \quad \mathrm{H} = \frac{\cdot 03395\, c\, \rho\, \Delta}{3\, e\, e'\, a^{\mathrm{T}_1}} = \frac{\cdot 03395 \times \cdot 0333 \times 0 \cdot 3 \times 13 \cdot 596}{3 \times 0 \cdot 8 \times 1 \times 1 \cdot 0077^{273}}$$

or $\mathrm{H} = \cdot 000237$.

102. From this M. Sarrau concludes, that it is not improbable that a reduction of temperature approaching to 2000° may take place in a close vessel, and that except for the cooling influence of the vessel, the theoretic temperature of 4080° might be realised.

But as has already been observed the value $c = 0\cdot185$ is probably too low for the specific heat of the products at the high temperature of combustion, and taking it at $0\cdot322$ as obtained in section (87), the value of z would be as shown in the following table, for cocoa and pebble powder.

	T, seconds.	w in kilogrammes.		
		0·1	0·5	1·0
		z	z	z
		°	°	°
Cocoa	·001	117	41	25
	·010	296	164	116
	·100	517	360	296
Pebble	·001	90	29	16
	·010	255	132	90
	·100	473	318	254

From this table it is seen, that the loss of temperature in $\frac{1}{1000}$ of a second when 1 kilog. of pebble burnt in 1 decimetre cube or with gravimetric density = 1, would be only 16° C. which corresponds to $16 \times \cdot332 = 5\cdot32$ units of heat, or about $\frac{1}{136}$ part of the heat evolved.

103. In the above calculation the fall of temperature z is obtained from the formula $z\,a^z = \dfrac{\mathrm{H}\,e\,e'\,\tau\,a^{\mathrm{T}_0}}{c} \cdot \dfrac{\sigma}{w}$ which gives

$$\log z + z \log a = \log \frac{\mathrm{H}\,e\,e'\,\tau\,a^{\mathrm{T}_0}}{c} + \log \frac{\sigma}{w},$$

and in the case of cocoa brown prismatic with $\tau = \cdot 001$,

$$\log z + z \times \cdot 0033 = \cdot 67632 + \log \frac{\sigma}{w}.$$

In the case above, if 1 kilog. is burnt in 1 decimetre cube $w = 1$ and $\sigma = 6$, which gives

$$z + \frac{z}{300} = 1 \cdot 45457, \text{ and } z = 25^\circ;$$

but in the case of a 12-inch gun firing 295 lb. of powder in a chamber whose surface is 185·6 decimetres we would have $\sigma = 185 \cdot 6$ decimetres and $w = 133 \cdot 8$ kilog.
Therefore

$$\log \frac{\sigma}{w} = \log \frac{185 \cdot 6}{133 \cdot 8} = 0 \cdot 14206.$$

Consequently

$$\log z + \frac{z}{300} = \cdot 67632 + \cdot 14206 = \cdot 81238,$$

from which z is found $= 6^\circ \cdot 45$ which corresponds to a loss of 2·15 units per kilogramme of powder or only about $\frac{1}{390}$ part of the heat evolved.

104. The above calculations are based on the assumptions, of a uniform rate of burning, that Dulong's law applies to the very high temperature dealt with, and that the value of ϵ which is unknown is equal to unity, the first of which is certainly not true and the other two uncertain, yet one or two important conclusions may be drawn from the results arrived at.

(*a*) That the temperatures of combustion existing in a gun are certainly far below those given in the first table (§ 84), and are probably more nearly represented by table (§ 87).

(*b*) That the approximate mean value of the specific heat of the products of combustion, at the temperature of combustion, is not far from 0·322.

(*c*) That the units of heat abstracted by the walls of the gun chamber are very insignificant, not exceeding $\frac{1}{136}$ part of the heat evolved in a small gun with pebble powder, and $\frac{1}{390}$

part in a 12-inch chambered gun with cocoa powder, in each case during $\frac{1}{1000}$ of a second.

105. It is therefore probable that the temperatures of combustion given in table (§ 87) may be adopted in future calculations, and that the effect of cooling, especially in large guns, may be neglected. This investigation therefore, however tedious it may thought to be, gives results of considerable value.

Tension of Gases.

106. Messrs. Noble and Abel adopt the hypothesis, that at the moment of combustion, a portion of the products amounting to about $0 \cdot 57$ of the total weight, are not gasified, but diffused in a liquid form throughout the gases, and that they remain throughout at the same temperature as the gases, and, even if it be admitted that this portion assumes a gaseous form at extreme temperatures, they maintain with Bunsen and Schischkoff, that the tension of such gases is too feeble to exert any sensible influence on the pressure.

They also maintain, that the pressure of the permanent gases may be calculated by the ordinary laws for gaseous matter, deducting from the volume of the containing vessel that of the non-gaseous matter at the temperature of combustion.

Let then

C = volume of containing vessel.

w = weight of powder.

T_0 = absolute temperature of combustion.

a = volume of non-gaseous matter arising from unit of weight of powder at temperature T_0.

v_0 = volume of the permanent gases arising from unit of weight of powder at temperature zero and atmospheric pressure.

p_0 = atmospheric pressure ($= 103 \cdot 33$ kilog. per square decimetre).

p = pressure after explosion.

Then $C - aw$ = vol. of permanent gases at T_0,

E

and $\frac{p}{p_0} = \frac{w v_0}{C - a w} \cdot \frac{T_0}{273}$ (adopting the Centigrade scale) and if we write $f = \frac{p_0 v_0 T_0}{273}$

$$p = f \frac{w}{C - a w}. \qquad (1)$$

107. If the vessel be not entirely filled with powder, the density of charge (gravimetric density) must be taken into account. Let this be represented by Δ, then $\Delta = \frac{w}{C}$,* and (1) becomes

$$p = f \cdot \frac{\Delta}{1 - a\Delta}.$$

108. The symbol f is what is called by M. Sarrau the "force" of the powder, but it must not be confounded with the absolute pressure or tension of powder exploded in a close vessel originally filled with powder, and which was determined by Noble and Abel to be about 43 tons per sq. inch or 6568 kilog. per sq. centimetre.

By "force" of the powder M. Sarrau denotes the pressure of the permanent gases arising from $\cdot 1$ kilog. of powder at the temperature T_0 of Combustion when occupying unity, i. e. 1 decimetre cube or 1 litre of space, that is to say, when exploded in a vessel whose capacity is $1 + a$, a being the volume occupied by the non-gaseous matter.

The value of f is $\frac{p_0 v_0 T_0}{273}$ and for the same powder $\frac{p_0 v_0 T_0}{273}$ is assumed to be constant, and independent on the gravimetric density of the charge.

v_0 is the volume of permanent gases at temperature zero, and in powders of the same composition may be considered as constant. In order that T_0 should be constant, it is necessary, first, that the exterior work be nil, which is of course

* Here the unities are the kilogramme and decimetre, and since 1 kilog. of powder occupies 1 dm.^{3}, $\Delta = \frac{w}{C}$.

the case in burning powder in a close vessel; second, that the cooling during the period of combustion is insignificant, which we have already shown to be the case; and third, that the interior work corresponding to the expansion of the gas shall be nil, which is in fact only a consequence of the assumption that the permanent gases follow the same law as a perfect gas.

It may therefore be assumed that the value of f is constant for each powder, and independent on the circumstances under which it is burnt. It may therefore be considered as a "characteristic" of the powder, directly proportionate to the tension or pressure, and may be called the actual strength of the powder.

109. From the formulæ $f = \frac{p_0 v_0 T_0}{273}$ and $p = f \cdot \frac{\Delta}{1 - a\Delta}$ the values of f and p may be calculated from the data contained in tables (§§ 79 and 87), making $a = 0 \cdot 57$, according to Noble and Abel's second paper,* and $\Delta = 1$.

This gives the following table:—

Description of Powder.	f Kilog. per square decimetre.	p Kilog. per square decimetre.	p Tons per square inch.	Potential metre-tons per kilog.
Cocoa—Brown Prismatic ..	200,700	466,700	29·62	365
Spanish pellet	231,100	537,400	34·12	335
Curtis and Harvey No. 6 ..	237,800	553,000	35·21	333
Waltham Abbey F.G. ..	248,700	578,200	36·71	322
" R.L.G. ..	258,450	601,000	38·16	317
" Pebble ..	263,600	612,800	38·93	314
Mining powder	262,700	610,800	38·80	225

110. In the last column of the above table, are placed the potentials of the powders (§ 11). It is curious to observe that the actual effect of the powder does not at all follow its potential. Comparing the cocoa with the mining powder the potential is 62 per cent. greater whilst the actual effect is 24½ per cent. less.

Comparing again the pebble and the mining powder the

* Phil. Trans., 1880. The value of a adopted by M. Sarrau was 0·60.

potential is nearly 40 per cent. greater, whilst the actual effect is nearly the same.

111. This last remark was confirmed by actual experiment by Messrs. Noble and Abel, who found the actual pressure fired in a close vessel to be

Waltham Abbey powder,	43 tons per square inch;		
Mining powder	44	„	„

and from this they conclude that the capacity for performing work of the various descriptions of powder is not very different. This conclusion is not borne out by the calculations, the results of which are given in the last table.

It is true that cocoa powder was not in use at the time of these experiments, but excluding it, there appears to be a difference between Spanish pellet and mining powder of about 12 per cent. in favour of the latter, whilst the potential was 50 per cent. greater in the former.

112. If the above results are even only approximately true, the disadvantages of cocoa powder seem apparent: not only is it a very weak powder, involving much heavier charges for equal ballistic effect, but it is a much more expensive powder and more difficult to manufacture.

In the evidence given by the Superintendent of the Royal Gun Factory before Lord Morley's Committee in 1887, it was stated that the costs were as follows:—

	s.	*d.*
Pebble manufactured at Waltham Abbey	46	4
„ bought from the trade	60	0
Cocoa prism, brown, Waltham Abbey ..	85	7
„ Westphalia Company	109	7½
„ Rothweill Company	110	5
„ Chilworth Company	123	9

113. When it is borne in mind that the cocoa powder is 24 per cent. less powerful than the pebble, it is apparent how disadvantageous it must be in an economical point of view. But the disadvantage does not stop here. The increased bulk of the charges, involves larger chambers in the guns, larger cartridges and larger magazines. Nor is this all, the increased bulk of the products of combustion,

and the increased temperature of combustion, must increase the erosion of the bore, and it is probable that much of the erosion which is becoming so very serious in our modern artillery, is due to these causes.

The presumed advantages of cocoa powder, viz. the low pressures obtained, are due chiefly, if not entirely, to the small surface and slow rate of ignition, and the consequent displacement of the projectile before the whole charge is consumed, thus keeping down the maximum pressure in the gun.

114. There is a difference of opinion among artillerists, with respect to the action of the products of combustion whilst expanding in a gun.

The first hypothesis, which is that of Messrs. Noble and Abel, is that the non-gaseous portion of the charge in a liquid state is very finely diffused throughout the gases, and is always at their temperature, thus giving out heat whilst they expand, and consequently to that extent preventing the fall of pressure.

The second hypothesis, that of Messrs. Bunsen and Schischkoff, is that the temperature of the non-gaseous portions remains constant or nearly so, and that whatever heat it gives off is simply radiated to the walls of the chamber, without affecting the temperature of the gases.

1*st Hypothesis.*

115. In a lecture read by Captain Noble on 3rd April, 1884, at the Institution of Civil Engineers, on "Heat Action of Explosives," he says, "In the researches made by Sir F. Abel and myself, when we found that the pressures in the bores of guns, and the energies generated by gunpowder, were far in excess of those deduced from Bunsen and Schischkoff's theory, we came to the conclusion that this difference was due to the heat stored up in the solid, or rather the liquid products of combustion. In fact these products, forming as they do nearly three-fifths of the weight of the powder, being also in a state of very minute division, constitute a source of heat of a very perfect character, and are

available for compensating the cooling effect due to the expansion of the gases on the production of work.

116. Captain Noble does not state in what way the great excesses spoken of were determined, but it is to be presumed that he is comparing the actual results obtained from the energy imparted to the projectile, with the results which would be deduced from a pressure curve formed on Bunsen and Schischkoff's theory. If so I am quite unable to accept his conclusions. The energy as measured from pressure curves, is always greater than the actual energy imparted to the projectile, as various other resistances have to be overcome, as will be seen hereafter, and unless it can be shown that these resistances are greater than the above differences, there is no need to seek for a source of heat in the non-gaseous products.

117. M. Sarrau has investigated this point by calculating the ballistic result, according to the two hypotheses in three different guns, and comparing these with the actual results obtained by firing.

The following is his method of procedure :—

1st Hypothesis.

118. Let W = exterior work done.

E = mechanical equivalent of heat.

c = mean specific heat of products of combustion at constant volume.

$y = F(t)$ = weight of these products at any time t.

T = their absolute temperature at same time.

T_0 = initial "absolute" temperature of combustion.

p_0 = atmospheric pressure.

p = pressure of gases at time t.

v_0 = specific volume of gases from unit of weight (1 kilog.) of powder at zero and atmospheric pressure.

V = volume of the gases at t.

Then

$$\frac{W}{E} = c\,y\,(T_0 - T); \tag{1}$$

but

$$p = \frac{p_0\,v_0\,y\,T}{273\,V}. \tag{2}$$

Eliminating T between (1) and (2) and observing that $f = \frac{p_0\,v_0\,T_0}{273}$ and writing $2\,\theta$ for $\frac{p_0\,v_0}{273\,E\,c}$, we get

$$p\,v + 2\,\theta\,W = f\,y. \tag{3}$$

Let then m = mass of the projectile.

ω = area of bore or transverse section of projectile.

l = distance moved by projectile at t.

z = length of initial void, or length of bore which would give the same capacity behind the projectile as the vacant space before the projectile began to move and consequently

$$V = \omega\,(z + l). \tag{4}$$

V is thus made up of

(*a*) V_0 = initial space in the chamber not filled with the charge.

(*b*) y_1 = interstices between the grains of powder which are unburnt at t.

(*c*) $\omega\,l$ = the cylindric volume generated by the motion of the projectile at time t.

When the whole of the charge is burnt there should be added, the original volume of the powder, less the volume of the residue of non-gaseous matter at the temperature of combustion, but as this is practically equal to the original volume of the powder, this item is reduced to zero.

Consequently

$$V = V_0 + y_1 + \omega\,l;$$

but, as before,

$$V = \omega (l + z).$$

Therefore

$$\omega z = V_0 + y_1 \quad \text{and} \quad z = \frac{1}{\omega}(V_0 + y_1). \tag{5}$$

Now supposing the ignition and combustion to be instantaneous, i. e. before the projectile moves, $V_0 + y_1$ is constant and therefore z is constant in this case.

Calling z_0 this particular value of z, its value is

$$z_0 = l_0 \frac{\delta - \Delta}{\delta} = l_0 \left(1 - \frac{\Delta}{\delta}\right) = \frac{w}{\omega}\left(\frac{1}{\Delta} - \frac{1}{\delta}\right) \tag{6}$$

in which l_0 is the length of a cylinder whose diameter is the same as the bore ω, and whose capacity is the same as that of the chamber. It may be called the "Reduced Length" or the "Equivalent Length" of the Chamber.

Δ is the gravimetric density ("Densité de Chargement").

δ, the absolute density of the powder.

w, weight of the charge.

Unities, kilogrammes and decimetres.

The equation of motion of the projectile is

$$(l + z)\frac{d^2 l}{d t^2} + \theta \left(\frac{dl}{dt}\right)^2 = \frac{fy}{m}. \tag{7}$$

119. On the hypothesis of combustion before the projectile moves, this equation is immediately integrable, and the resulting velocity, though not exact, will be the superior limit of the realisable velocity.

Therefore

$$v^2 = \frac{fw}{\theta m}\left\{1 - \left(\frac{z_0}{l + z_0}\right)^{2\theta}\right\}. \tag{8}$$

Now, as was shown above,

$$f = \frac{p_0 v_0 T_0}{273} \quad \text{and} \quad \theta = \frac{p_0 v_0}{2 \times 273 \times E c};$$

therefore

$$\frac{f}{\theta} = 2 E c T_0$$

and

$$v^2 = 2\,\mathrm{E}\,c\,\mathrm{T}_0 \,\frac{w}{m}\left\{1 - \left(\frac{z_0}{l + z_0}\right)^{2\theta}\right\}$$
$$= 2\,\mathrm{E}\,\mathrm{Q}\,\frac{w}{m}\left\{1 - \left(\frac{z_0}{l + z_0}\right)^{2\theta}\right\}. \qquad (9)$$

The values of θ for the powder used by M. Sarrau were

	dm.			
For the small gun	0·65	calibre,	sporting powder	0·0549
gun	1·53	,,	cannon	0·0612
gun	2·42	,,	Wetteren $\frac{13}{16}$ ..	0·0657

2nd Hypothesis.

120. By this hypothesis the non-gaseous matter imparts no heat to the gases.

Let then ϵ = weight of non-gaseous products from 1 kilog. and T, T_0, and y = as before.

c_1 = specific heat at constant volume of gaseous products.

Then, as before,

$$\frac{\mathrm{W}}{\mathrm{E}} = c_1\,\epsilon\,y\,(\mathrm{T}_0 - \mathrm{T}) \qquad (10)$$

which only differs from (1) by changing $c_1\,\epsilon$ to c.

Therefore,

$$2\,\theta = \frac{p_0\,v_0}{273\,\mathrm{E}\,\epsilon\,c_1}; \qquad (11)$$

and since v_0 is the volume of a weight of gas ϵ, the specific volume $= \frac{v_0}{\epsilon}$; and if c^1 = specific heat at constant pressure

$$\mathrm{E} = \frac{p_0\,v_0}{273\,\epsilon\,(c^1 - c_1)};$$

and if

$$n = \frac{c_1'}{c_1},$$

and

$$\frac{c^1}{c_1} = 1\cdot 4$$

$$\theta = \frac{n-1}{2} = 0\cdot 2.$$

Now by (11)

$$\theta = \frac{p_0 v_0}{2 \times 273 \mathrm{E} \epsilon c_1}$$

$$\therefore \quad \frac{f}{\theta} = 2 \epsilon c_1 \mathrm{E} \mathrm{T}_0 ;$$

or if

$$\mathrm{Q} = c \mathrm{T}_0 ,$$

$$\frac{f}{\theta} = 2 \epsilon \frac{c_1}{c} \mathrm{E} \mathrm{Q} ,$$

which values inserted in (8) give

$$v^2 = 2 \epsilon \frac{c_1}{c} \mathrm{E} \mathrm{Q} \frac{w}{m} \left\{ 1 - \left(\frac{z_0}{l + z_0} \right)^{\frac{2}{\cdot 5}} \right\} \quad (12)$$

or

$$v^2 = 2 \epsilon c_1 \mathrm{E} \mathrm{T}_0 \frac{w}{m} \left\{ 1 - \left(\frac{z_0}{l + z^0} \right)^{\cdot 4} \right\} . \quad (12)$$

121. The equations are therefore,

1st Hypothesis, $v^2 = 2 \mathrm{E} \mathrm{T}_0 c \frac{w}{m} \left\{ 1 - \left(\frac{z_0}{l + z_0} \right)^{0 \cdot 6814} \right\}$ (13)

2nd Hypothesis, $v^2 = 2 \mathrm{E} \mathrm{T}_0 \,.\, c_1 \epsilon \,.\, \frac{w}{m} \left\{ 1 - \left(\frac{z_0}{l + z_0} \right)^{\cdot 4} \right\}$ (14)

for Wetteren powder, in both cases.

The values of c and c_1 used by Sarrau are those given by Bunsen and Schischkoff, viz.

c = mean specific heat of all products .. .. $\cdot 185$
c_1 = specific heat of gases only .. .. .. $\cdot 164$

therefore

$$\frac{c_1}{c} = 0 \cdot 887 .$$

The specific heats as determined by Noble and Abel for pebble powder are

$$c = 0 \cdot 1916$$
$$c_1 = 0 \cdot 1760 ,$$

therefore

$$\frac{c_1}{c} = 0 \cdot 918 .$$

122. From the above formula M. Sarrau calculated the following table, giving the calculated as compared with the actual results obtained by firing with three guns.

		dm.		
1.	Military rifle	0·175	calibre,	fired with sporting powder.
2.	Rifled gun	1·53	„	ordinary cannon powder.
3.	„	2·42	„	Wetteren $\frac{13}{16}$.

The table also gives the Ballistic Elements of the Guns.

123.

Designation of Ballistic Elements. Kilogrammes and decimetres.	Calibre.		
	0·175	1·53	2·42
c = Calibre	·175	1·53	2·42
l = Travel of projectile	9·80	27·91	34·45
l_0 = Equivalent length of chamber	..	2·95	7·63
Δ = Gravimetric density	1·00	·537	·798
z_0 = Reduced length of initial void	·0435	1·92	4·24
w = Weight of charge	·0025	2·90	28·00
W = „ projectile = mg.	·036	24·00	144·00
Q = Heat of combustion = $c\,T_0$	·849	·795	·795
ϵ = Weight of gaseous products of 1 kilogramme	·337	·412	·412
V_{13} = Velocity from (13)	4940	4820	5600
V_{14} = „ (14)	3650	4470	5330
V_0 = Observed velocity	2750	3340	4320

124. It will be observed, that in all cases the calculated velocities are considerably greater than the observed velocities. In the case of the 1·53 gun the velocity given by (13) is about 45 per cent., and in the 2·42 gun about 30 per cent. above the observed velocity, whilst by formula (14) the excesses are 34 per cent. and 24 per cent. respectively.

Now, unless the passive resistances ~~equal~~ are greater than this 24 per cent., there is no reason why an increase of temperature should be sought for from the non-gaseous products according to Noble and Abel's hypothesis, and although the above table is only a rough approximation, it represents pretty accurately the comparative results of the two hypotheses.

As regards the two guns of 1·53 and 2·42 calibre, the

excess of velocity due to the excess heat imparted to the gaseous products by Noble and Abel's hypothesis is about 7·8 per cent. and $5\frac{1}{4}$ per cent. respectively.

On the Movement of the Products of Combustion.

125. It is usually assumed that for the real state of the products of Combustion, an ideal state may be substituted; in which the whole mass should at each instant have a uniform density and temperature equal to the mean density and temperature of the mixture, and on this hypothesis the movement of the products has been analysed from the principles of the motion of permanent gases.

126. M. Sarrau treats this question as follows:—

Let y = weight of products of Combustion at any time t.

W = exterior work done.

μ = total mass of the charge $= \frac{w}{g}$.

$\delta\mu$ = mass of an element of the mixture of gaseous and non-gaseous and unconsumed portions of the charge at the time t.

x = distance of $\delta\mu$ from breach at t.

T = mean temperature of products of Combustion.

c = specific heat of products at constant volume.

T_0 = "absolute" temperature of Combustion.

127. If the products of Combustion were formed without any sensible internal movements of their own and without production of external work, they would remain at the temperature T_0. But by reason of their proper motion and the work done in expanding in the bore, the temperature falls from T_0 to T, and the work due to this fall of temperature is equal to half the vis viva of the charge plus the external work done.

Consequently equation (1), p. 55, $W = E\,c\,y\,(T_0 - T)$ becomes

$$W + \frac{1}{2}\int\left(\frac{d\,x}{d\,t}\right)^2 \delta\mu = E\,c\,y\,(T_0 - T). \qquad (15)$$

Now by (2)

$$p = \frac{p_0 v_0 y \mathrm{T}}{273 \mathrm{V}} \quad \text{or} \quad p \mathrm{V} = \frac{p_0 v_0 y \mathrm{T}}{273}$$

becomes

$$p \mathrm{V} = \frac{p_0 v_0}{273} . y \mathrm{T} - \int x \frac{d^2 x}{d t^2} \delta \mu. \tag{16}$$

Eliminating y T between (15) and (2)

$$p \mathrm{V} + \cancel{x} \int x \frac{d^2 x}{d t^2} \delta \mu + 2 \theta (\mathrm{W} + \tfrac{1}{2} \int \left(\frac{d x}{d t}\right)^2 \delta \mu) = f y. \tag{17}$$

128. Now let $l + l_0$ be the distance of the base of projectile from the breech, l_0 being the original distance or equivalent distance, and l the travel of projectile. The distance x is evidently less than $l + l_0$, and the velocity and acceleration of the element $\delta \mu$ less than those of the projectile, therefore,

$$\int x \frac{d^2 x}{d t^2} \delta \mu = a \mu (l + l_0) \frac{d^2 l}{d t^2} \tag{18}$$

and

$$\int \left(\frac{d x}{d t}\right)^2 \delta \mu = a_1 \mu \left(\frac{d l}{d t}\right)^2, \tag{19}$$

a and a_1 being coefficients less than unity.

129. If m be the mass of the projectile,

z, the reduced length of the initial void equal to

$$l_0 \left(1 - \frac{\Delta}{\delta}\right)$$

$$p \mathrm{V} = m (l + z) \frac{d^2 l}{d t^2}, \tag{20}$$

$$\mathrm{W} = \tfrac{1}{2} m \left(\frac{d l}{d t}\right)^2, \tag{21}$$

and writing β for $\dfrac{l + l_0}{l + z}$,

(18) becomes
$$\int x \frac{d^2 x}{d t^2} \delta \mu = a \beta \mu (l + z) \frac{d^2 l}{d t^2}, \tag{22}$$

and (17) becomes

$$(l + z) \frac{d^2 l}{d t^2} \left(1 + a \beta \frac{\mu}{m} \right) + \theta \left(\frac{d l}{d t}\right)^2 \left(1 + a_1 \frac{\mu}{m}\right) = \frac{f y}{m}, \tag{23}$$

which is the equation of motion of the projectile, and differs from that previously obtained (7) only by the coefficients of the two terms of the first member.

130. These coefficients reduce sensibly to unity when the mass of the charge is small compared with that of the projectile. This condition, however, is not usually realised in practice as the ratio $\frac{\mu}{m}$ is often $\frac{1}{3}$ or $\frac{1}{2}$, and the coefficients therefore vary considerably from unity.

To ascertain their actual value it would be necessary to know the law of variation of the velocities and various accelerations of the elements of the charge. This law has been the subject of many researches by Lagrange, Poisson, and Piobert, but the problem is a very difficult one, and the solutions too complex and the bases too uncertain to make it of practical utility.

131. M. Sarrau however remarks, that happily its importance is only secondary, and that the equation (7) really renders account of the most essential elements of the problem.

In this point of view he remarks, that a and a_1 have values less than if the whole charge were instantaneously gasified. At any given moment of time, the non-gaseous products and the unburnt gases are diffused throughout the whole mass of the gases, and so surrounded by them that the pressure on the whole surface of each particle is practically equal. Their accelerations and velocities are therefore very small, consequently the corresponding elements of the integrals in (18) and (19) would have an insensible value, and consequently the integrals are diminished. This, however, would not be so, if the time of Ignition was greater than that of the combustion of the grain, which might be the case if the grain of the powder was small and the length of the charge very great, in which case the front part of the charge might be driven forward with the velocity of the projectile, or even less, wedging the powder in the front part of the chamber.

132. If the whole of the charge be consumed when the

projectile begins to move the accelerations and velocities of the mixture of products may be determined as follows :—

Let the mass of products be divided into an infinite number of thin slices at right angles to the axis of the bore, and suppose the whole of the particles of each slice to have the same velocity and acceleration parallel to the axis.

Let ρ = the density of any one slice ;

dx = its thickness ;

ω = area of the bore ;

then $\rho \omega dx$ is the mass of the slice and (18) and (19) become

$$\omega \int x \frac{d^2 x}{d t^2} \rho \, d x,$$

and

$$\omega \int \left(\frac{dx}{dt}\right)^2 \rho \, d x;$$

to integrate which it is necessary to know the law connecting the velocity and accelerations with the position of the slice.

134. Piobert supposed the velocity of any slice to be proportional to its distance from the end of the bore, or

$$\frac{dx}{dt} = \frac{x}{l + l_0} \cdot \frac{dl}{dt}. \tag{24}$$

Differentiating with respect to t,

$$\frac{d^2 x}{d t^2} = \frac{1}{l + l_0} \cdot \frac{dx}{dt}\frac{dl}{dt} + \frac{x}{l + l_0} \cdot \frac{d^2 l}{d t^2} - \frac{x}{(l + l_0)^2} - \left(\frac{dl}{dt}\right)^2$$

and replacing $\frac{dx}{dt}$ by its value from (24),

$$\frac{d^2 x}{d t^2} = \frac{x}{l + l_0} \cdot \frac{d^2 l}{d t^2}, \tag{25}$$

from which it appears that the law of the accelerations is the same as that of the velocities.

135. The density ρ varies throughout the mass according to an unknown law, but the variation is probably not great,

and may be neglected in the calculation of terms whose numerical importance is relatively small.

Under this hypothesis

$$\int x \frac{d^2 x}{d t^2} \delta \mu = \frac{\rho \omega}{l + l_0} \frac{d^2 l}{d t^2} \int x^2 d x$$

and

$$\int \left(\frac{d x}{d t}\right)^2 \delta \mu = \frac{\rho \omega}{(l + l_0)^2} \left(\frac{d l}{d t}\right)^2 \int x^2 d x.$$

The integral $\int x^2 d x$ is to be taken within the limits $l + l_0$ and 0 and its value is therefore $\frac{(l + l_0)^3}{3}$. Substituting which, and observing that

$$\rho \omega (l + l_0) = \mu,$$

we get

$$\int x \frac{d^2 x}{d t^2} \delta \mu = \frac{1}{3} \mu (l + l_0) \frac{d^2 l}{d t^2}$$

and

$$\int \left(\frac{d x}{d t}\right)^2 \delta \mu = \frac{1}{3} \mu \left(\frac{d l}{d t}\right)^2;$$

but by (18) and (19)

$$\int x \frac{d^2 x}{d t^2} \delta \mu = a \mu (l + l_0) \frac{d^2 l}{d t^2},$$

and

$$\int \left(\frac{d x}{d t}\right)^2 \delta \mu = a_1 \mu \left(\frac{d l}{d t}\right)^2$$

therefore we get

$$a = a_1 = \frac{1}{3}. \tag{26}$$

This value is the superior limit, and as only about $\frac{4}{10}$ths of the charge is reduced to gas, the real value may not exceed $\frac{4}{30}$.

136. When the reduction to gas is total, the coefficient $\beta = \frac{l + l_0}{l + z}$ is reduced to unity.

When the reduction is partial and progressive β approaches

to unity with increasing values of l, and may still be considered as $= 1$ without inconvenience.

When l is very small, β differs from unity, but in this case a and a_1 are very small, and the influence of these terms is negligible.

137. If in (23) $a = a_1$ be made $\frac{1}{3}$ we get

$$(l + z) \frac{d^2 l}{d t^2} + \theta \left(\frac{d l}{d t}\right)^2 = f y \frac{1}{m \left(1 + \frac{1}{3} \frac{\mu}{m}\right)} \tag{27}$$

so that the essential form of (3) remains the same, there being a slight change in the second side only, and this may be treated as an increase of the mass of the projectile by $\frac{\mu}{3 m}$.

If $\frac{\mu}{m} = \frac{1}{3}$ the mass of the projectile may be considered as increased by one-ninth.

M. Sarrau points out, that within the usual limits of practice, the ratio $\frac{\mu}{m}$ does not vary much, and in such case the effect of its introduction in (27) is virtually to reduce the value of f.

In like manner, he says that the effect of cooling by transmission of heat to the metal of the gun, may be taken into account in estimating the value of f. This is no donbt true, and it can lead to very small error, provided the value of f as estimated is applied to ballistic elements not too widely differing from those of its estimation.

138. It is very truly observed by M. Sarrau, that the *a priori* determination of f, by the methods above given, almost loses its practical value, inasmuch as we are not in a position to apply to it the proper reductions due to the various causes of loss of work.

On the other hand, too much importance must not be attached to this. He says, "It would be very difficult, and

of no great practical utility to obtain a formula which would permit the calculation *a priori* of all the effects, without borrowing from experiment certain practical data; but a theory which by obtaining from a few practical experiments certain coefficients, and which would then serve to calculate the effects realisable in other conditions than those of the actual experiments, would evidently be of no great practical use. What is really required is to establish the *form* of the relations which bind together the effects obtained, to the "Ballistic Elements," and thus to avoid the use of *purely empirical* formulæ, which in the absence of any natural guidance, may be found incompatible with the nature of the phenomena which they are intended to represent.

139. The formulæ generally given in treatises of Ballistics are purely empirical; they embody the laws which combine together the velocity of the projectile with certain ballistic elements, such as the weights of charges and projectiles, the length and calibre of the guns, but they do not take account in any way of the peculiar properties of the powder itself.

It is here that M. Sarrau has stepped in and by the guidance of theoretical considerations, has succeeded in a very remarkable degree in obtaining formulæ for velocity and pressure, into which the special character and nature of the powder, its force, the rate of its combustion, and the form of the grain are all introduced, and thus the discussion of these formulæ has led to very important conclusions relatively to the conditions of loading, and the nature of the powder suitable in every case.

The following chapter is devoted to an explanation of the methods followed by M. Sarrau, and it is indeed little more than an abstract of his own writings, which, by his kind liberality I am permitted to make use of.

CHAPTER III.

M. SARRAU'S FORMULÆ.

140. In the establishment of formulæ for the initial velocity and pressure, M. Sarrau commences by seeking from theoretical considerations, the *rational*, if not the *exact form* of the relations which bind these results to the ballistic elements (Éléments du Tir) such as the weight of charge and projectile, gravimetric density, and dimensions of the gun. Then he determines by experiment the value of the constants which enter into these formulæ, and finally he verifies the results by a comparison with actual practice. His formulæ contain certain factors which he terms "characteristics of the powder," and which depend upon its composition, the form and size of grain, and the velocity of burning in free air.

141. He assumes,

(*a*) That the period of Ignition of the charge is small in comparison with that of its Combustion, an assumption which is no doubt correct with regard to modern powders such as Pebble and upwards in size, and which may be made true with small grain powders by proper conditions of loading, but which probably is less true with prismatic powders packed closely together, so as to prevent to a great extent the immediate access of the Ignition to the outer surface of the grains.

(*b*) That the permanent gases of the products of combustion in expanding, do not receive any heat from the non-gaseous products. This is contrary to the opinion of Messrs. Noble and Abel, but it is supported by the fact that the permanent gases are very diathermal and have but a

feeble absorbing power, and, as has been previously shown, there is no deficiency in heat, when the actual results are compared with calculation, to necessitate any such collateral source of heat as is supposed by Noble and Abel.

Differential Equation of Motion of Projectile.

142. Let q = weight of powder burnt at the end of time t.

x = distance passed through by projectile at do.

P_1 = mean pressure of gases in kilog. per sq. metre.

V_1 = volume of the space behind the projectile less the original volume of the charge in cubic metres.

v = velocity of projectile.

143. If at the time t, the charge ceased to burn, the gases already formed would continue to expand adiabatically, and if P be the mean pressure and V the volume at a time t_1 greater than t

$$P\,V^n = P_1\,V_1^n \qquad (1)$$

where n is the ratio of specific heat at constant pressure to that at constant volume $= \frac{C_p}{C_v}$* $= \frac{\cdot 2324}{\cdot 1762} = 1\cdot 319$.

Let W = exterior work done at the time t_1, then the work done at $d\,t_1$ will be $d.W$, and

$$d\,.\,W = P\,d\,V. \qquad (2)$$

Introducing the value of P from (1) and integrating

$$W = -\frac{P_1\,V_1^n}{n-1}\,V^{1-n} + C, \qquad (3)$$

but when $t = t_1$, $V = V_1$, and $W = \frac{1}{2}\,m\,v^2$ (neglecting the vis viva of the charge), therefore

$$C = \tfrac{1}{2}\,m\,v^2 + \frac{P_1\,V_1}{n-1}$$

* Noble and Abel, 2nd Memoir.

and

$$W = -\frac{P_1 V_1^n}{n-1} V^{1-n} + \tfrac{1}{2} m v^2 + \frac{P_1 V_1}{n-1}. \qquad (4)$$

If now the gun be supposed indefinitely long, and V tends to infinity, W tends to $\frac{10fq}{n-1}$ and $\frac{P_1 V_1}{n-1} V^{1-n}$ tends to zero, since n is greater than unity.

Therefore

$$\frac{10fq}{n-1} = \frac{mv^2}{2} + \frac{P_1 V_1}{n-1}. \qquad (5)$$

144. That W tends to $\frac{10fq}{n-1}$ is thus shown.

q If w be the weight of a charge burnt, the gases evolved per unit of weight, will occupy at the origin a volume

$$v = \frac{V_0}{q}; \text{ therefore } P_0 V_0 = P_0 v\, q.$$

The product $P_0\, v$ is constant, whilst v varies, so long as the temperature T_0 is constant, but when $v = 0 \cdot 001$ cubic metre the pressure is what M. Sarrau calls the "force of the powder" and denotes by f.

If therefore f be expressed in kilogrammes per square centimetre, and P_0 in formula (4) in kilogrammes per square metre

$$P_0 v = 10{,}000 f \times \cdot 001 = 10 f;$$

or making $v = V_0$ $\frac{v_0}{q}$ $P_0 V_0 = 10 f w.$ q

But

$$W = \frac{P_0 V_0}{n-1} \left\{ 1 - \left(\frac{V_0}{V} \right)^{n-1} \right\}.$$

$$\therefore \quad W = \frac{10fq}{n-1} \left\{ 1 - \left(\frac{V_0}{V} \right)^{n-1} \right\}.$$

When the gun is prolonged indefinitely $\frac{V_0}{V}$ tends to zero, and therefore W tends to $\frac{10fq}{n-1}$ as stated in the preceding paragraph.

145. Equation (5) may be transposed thus :—

Since $v = \frac{dx}{dt}$, and $V_1 = \omega\,(x + z)$ and $P_1\,\omega$ differs very little from the moving force, or $m\,\frac{d\,x^2}{d\,t^2}$. $\frac{d^2y}{dt^2}$

Inserting these values in (5)

$$\frac{10\,f\,\varpi}{n-1} = \tfrac{1}{2}\,m\left(\frac{d\,x}{d\,t}\right)^2 + m\,\frac{d^2\,x}{d\,t^2}\cdot\frac{x+z}{n-1}\quad .\;.\;.\;.$$

or including the factor 10 in f, which is only changing the unity of measure of f, which then becomes hectogrammes per square centimetre,

$$\frac{f\,\varpi}{n-1} = \tfrac{1}{2}\,m\left(\frac{d\,x}{d\,t}\right)^2 + m\,\frac{d^2\,x}{d\,t^2}\cdot\frac{x+z}{n-1}\,. \tag{6}$$

146. This formula, which is the same as that arrived at by M. Sarrau, has been deduced by a somewhat shorter method by Captain Roulin of the French Artillery, from whom the above is borrowed.

It is not rigorously exact because in deducing it several elements of minor importance have been neglected, but as we proceed to show, all these may be taken into account in the factor f, which is a purely numerical factor ascertainable by experiment.

147. (*a*) The gases are to some extent cooled by contact with the walls of the chamber, and therefore the work done is less than $\frac{f\,\varpi}{n-1}$ by a quantity depending on this cooling action, and this quantity is one depending on the weight of powder burnt as compared with the amount of cooling surface. Were this determined, the error might be compensated by altering the value of f.

(*b*) At any time t, the work done is not strictly $\frac{m\,v^2}{2}$, but half the vis viva of the whole system, including the projectile, the charge, the gun itself, and the carriage. Consequently the value of m is increased.

The real value of this term is in fact

$$\frac{m v^2}{2}\left\{1+\left(\frac{\rho}{\mathrm{R}}\tan\theta\right)^2+\frac{\mu}{3m}\right\}+\mathrm{M}\,v'^2$$

$$=\frac{m v^2}{2}\left\{1+\left(\frac{\rho}{\mathrm{R}}\tan\theta\right)^2+\frac{\mu}{3m}+\frac{\left(m+\frac{\mu}{2}\right)^2}{\mathrm{M}\;m}\right\}.$$

where

M = mass of gun and carriage.
μ = mass of the charge.
ρ = radius of gyration of projectile.
R = radius of the bore.
θ = angle of rifling.
v' = velocity of recoil.

The value of the factor within the brackets is generally not much greater than unity, because generally

$$\frac{\rho}{\mathrm{R}}\tan\theta < \cdot 005,$$

$$\frac{\mu}{3m} < \cdot 1666,$$

$$\frac{\left(m+\frac{\mu}{2}\right)^2}{\mathrm{M}+m} < \cdot 03,$$

and in any case these terms are independent of the time, and consequently may be made to enter into the value of m in $\frac{m v^2}{2}$.

(c) Since there are passive resistances opposed to the motion of the projectile, $\mathrm{P}\,\omega$ is not $= m\frac{d^2 x}{d t^2}$ but has a greater value, consequently this value of the second term is too small, and to make up for the passive resistances it is necessary to increase m in this term.

147a. Consequently, in order to make equation (6) correct it is necessary

1st. To diminish f on account of cooling.

2nd. To increase m in the two terms of the 2nd member, or what comes to nearly the same thing on account of the

form of the equation, diminish still further the value of f in the first member.

Hence all the corrections relative to the neglected elements may be included in the value of f, and this quantity is therefore simply a numerical coefficient, the value of which must be found by experiment.

148. The integration of (6) will give the form of the relation which unites the effects, viz. velocity and pressure, to the various and variable ballistic elements.

Writing $\theta = \frac{n-1}{2}$ equation (6) becomes

$$(x + z)\frac{d^2 x}{d t^2} + \theta\left(\frac{d x}{d t}\right)^2 = \frac{f q}{m}. \qquad (7)$$

θ being a constant, n was formerly estimated at 1·41, but the more recent estimate by Noble and Abel is 1·31.

148a. Before proceeding further it is necessary to make a few remarks respecting the Combustion of the charge.

149. Let w be the weight of the charge and q the weight of the powder burnt at the time t, then q is a function of t and may be represented by $w.\psi(t)$.

$\psi(t)$ depends on the form of grain and on the rate of burning of the powder in free air, and before proceeding with the integration of (7) it is necessary to determine the form of this function.

150. As was previously remarked, a distinction must be made between Ignition and Combustion, and in every case, if a high ballistic effect is sought for, ignition should be as nearly simultaneous as possible throughout the charge. With large-grained powders this is obtained by the frequency and size of the interstices between the grains, whilst with fine-grained powder it may be obtained by a hollow space such as a perforated tube extending throughout the full length of the cartridge. With very large charges it may happen that the Ignition of the front part of the charge takes place at so late a period that many of the grains leave the muzzle of the gun long before they are consumed. It is needless to say, that this is simply bad artillery practice and

it would ~~it~~ be folly to seek for any ballistic formula suitable for such cases; nor has this been attempted by M. Sarrau. He assumes that the period of Ignition is very small compared with that of Combustion, an assumption which I believe ought to be realised in all cases.

151. It is therefore with Combustion that we have now to deal.

Let then S be the ignited surface, which is supposed to be the whole surface of the grain;
V, the velocity of combustion at any time t;
δ, the absolute density of the powder;
η, the "rate of emission" of gas;
The volume burnt in $dt = S\,V\,dt$, and
The weight " $dt = S\,V\,\delta\,dt$.

Consequently the rate of emission, i. e. the volume evolved in $dt = \dfrac{S\,V\,\delta\,dt}{dt} = S\,V\,\delta$. Nonsense.

152. It was found by Piobert's experiments that the velocity of burning varied inversely as the density, consequently $V\delta$ is constant, and the rate of emission is proportional to the surface simply, but this surface is constantly varying and the variation depends on the form of the grain.

153. The case of Combustion in free air will first be considered. The grains being homogeneous and spherical, and the combustion being in free air, of course the pressure is constant.

Now the velocity of combustion and the density being constant, the velocity of emission is simply as the surface, and this surface is proportional to the square of the radius, and as the radius decreases uniformly, the emission of gas is inversely as the square of the time.

If R be the original radius, the original volume is $\dfrac{4\pi R^3}{3}$, and at the end of the time t, it is reduced to $\dfrac{4\pi}{3}(R - V\,t)^3$ and the weight of powder burnt will be

$$w_t = \frac{4\pi}{3}\delta\left\{R^3 - (R - V\,t)^3\right\}.$$

If τ be the total time of combustion of the grain

$$\tau V = R \quad \text{or} \quad V = \frac{R}{\tau},$$

then

$$w_t = \frac{4}{3}\pi\delta R^3\left\{1 - \left(1 - \frac{t}{\tau}\right)^3\right\} = w_0\left\{1 - \left(1 - \frac{t}{\tau}\right)^3\right\}$$

where w_0 is the original weight of the grain.

The "velocity of emission" at t is

$$\eta_t = \frac{dw}{dt} = \frac{w_0 . 3}{\tau}\left(1 - \frac{t}{\tau}\right)^2.$$

When the grains are not exactly spherical, nor of exactly the same dimensions, the same formula may be used, taking the radius of the mean spherical grain to calculate τ. If for instance N be the number of grains in 1 kilogramme of powder, R_m the radius of the mean spherical grain

$$\tfrac{4}{3}\pi R_m^3 \delta N = 1,$$

or

$$R_m = \left(\frac{3}{4\pi\delta N}\right)^{\frac{1}{3}}.$$

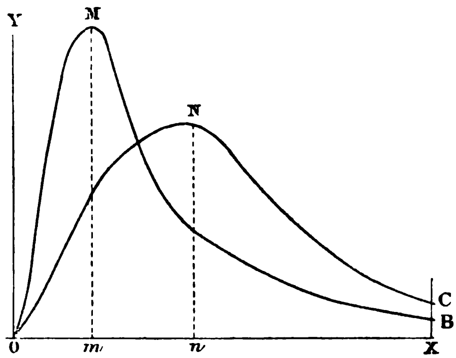
154. When powder is burnt in a gun, the same law of emission would hold if the velocity of combustion were uniform, but this is never the case in reality, since the velocity of combustion depends on the pressure. Now as the evolving gases are confined by the projectile, and as the emission of gas is very rapid at first on account of the large surface, it follows that the pressure rises rapidly at first, and this again by increasing the velocity of combustion, further increases the emission of gas, and the result is that before the projectile has moved far, a very high pressure is established, but as the projectile rapidly acquires velocity and increases the space behind it, the pressure begins to fall, and it falls rapidly, not only on account of the increasing space but on account of the loss of temperature in the gases due to the conversion of heat into energy, and also to the decrease of the velocity of emission due, both to the rapidly

decreasing surfaces and to the rapidly decreasing velocity of combustion caused by the decrease of pressure.

Consequently in a case like this, and especially when the grains are small and the total time of combustion also small, very high initial pressures are formed.

155. On the other hand, if it were possible to take a powder such that the initial velocity of emission were small and the subsequent rate of decrease also small or uniform, we would have a curve of pressure rising much less rapidly at first and falling less rapidly after the maximum.

In the first case the pressure curve may be represented by O M B and in a second by O N C. Now if with the same weight of powder the curve O N C X could be made equal to O M B X, there would be the same ballistic effect, whilst the strains on the gun would be less in the proportion of N n to



M m, but this can never be the case. For if the weight of powder and its composition be the same, and if we suppose the gun indefinitely prolonged, the two curves O M B and O N C both become asymptotic and as the heat developed is the same, the area of these curves must be the same.

But the ascending branch of the curve O N C is lower than that of O M B at first, it then crosses it and remains higher throughout the rest of its course; consequently the area of O N C beyond the ordinate X B C is greater than the

area of O M B beyond the same ordinate, and therefore the part O N C must be less than O M B.

From this it is evident that the so-called progressive powders, even if their manufacture were possible, must give a lower ballistic effect than ordinary powders, although of course they strain the gun less; but the same result may be accomplished much more easily by attending to the form and dimensions of grain.

156. When the size of grain is increased the initial surface *of the charge* and the rate of decrease of the velocity of emission in free air are both decreased.

For let there be two charges of the same weight of cubical grain powder, and let the length of the side be a and the number of grains N in one charge, and the length of the side $2a$ in the other, then the number of grains will be $\frac{N}{8}$.

Thus the surface of the first charge $= N \times 6a^2$, and of the second $\frac{N}{8} \times 6 \times (2a)^2 = \frac{N \times 6a^2}{2}$,

or just one-half of the surface of the first.

Also if V be the velocity of combustion in free air, then at the end of the time t, the dimensions of the grains of the first charge will be $a - 2Vt$, and of the second $2a - 2Vt$, and the total time of burning will be, for the first, $t = \frac{a}{2V}$ and for the second, $t_1 = \frac{a}{V} = 2t$.

Consequently the velocity of emission of the second charge is less at first, and decreases more slowly than in the case of the first charge with smaller grains.

157. The same result of decreasing the velocity of emission, might be attained by increasing the density, and as a matter of fact, the usual method of obtaining what are called slow powders is, both to increase the density and the size of grains. It must, however, be remembered that in decreasing the velocity of combustion and thus obtaining slow powders, we increase the time of burning of the charge, and may do so to such an extent that a considerable portion

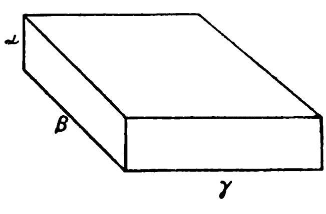
of it may be blown out unburnt. This of course necessitates an increase in the length of the gun, with all its attendant practical inconveniences.

158. An approach to uniformity of emission so far as that depends on surface, is made in so-called disc powders, that is to say in powders pressed into thin discs of uniform thickness, and of the same diameter as the gun chamber, so assembled together as to permit simultaneous ignition between adjacent discs. In this case the surface being nearly constant, the velocities of emission in free air would be nearly uniform and the time of total combustion dependent on the thickness of the discs.

Mr. Quick has patented this description of powder and is apparently getting very excellent results in a 4-inch gun. The difficulty that has been met with by others is that of the breaking up of the discs during combustion, thus giving rise to irregularity of inflamed surface and consequent irregularity of emission of gas. It is probable that this practical difficulty will be overcome, and that disc powder will be found very advantageous in guns of large as well as of small calibre.

159. In France an approach to the advantages of disc powder has for some years been obtained by the use of prismatic grains in which the thickness is small relative to the length and breadth.

For instance, let the grain be of the form shown in the



diagram, of the lineal dimensions α, β, and γ, of which α is the smallest. In this case the time of total combustion

$$T = \frac{\alpha}{2V},$$

If $S = 6 N a^2$ be the surface of the charge composed of cubical grains whose sides $= a$ this charge will be consumed in the same time $\tau = \dfrac{a}{2V}$.

For the flat grains the original surface of the same weight of charge is

$$S^1 = N \times \frac{a^3}{\alpha \beta \gamma} \times \{2 \alpha \beta + 2 \beta \gamma + 2 \gamma \alpha\} = 2 N a^2 \left(\frac{a}{\gamma} + 1 + \frac{a}{\beta}\right),$$

and making $\dfrac{a}{\beta} = x$ and $\dfrac{a}{\gamma} = y$ we get

$$S^1 = \frac{S}{3}(1 + x + y).$$

The surface at the end of the time T will be

$$S_1^1 = N \frac{a^3}{\alpha \beta \gamma} \times 2 \{(\beta - 2 V \tau)(\gamma - 2 V \tau)\};$$

or, since $\tau = \dfrac{a}{2V}$,

$$S_1^1 = 2 N \frac{a^2}{\beta \gamma}(\beta - a)(\gamma - a) = \frac{S}{3}(1 - x)(1 - y),$$

and since x and y are each less than unity we have S^1 less than S and S_1^1 greater than S; therefore with the flat grains the velocity of emission is less at the beginning, and greater at the end of the combustion, that is to say it decreases less rapidly.

It is therefore evidently advantageous to make the ratios x and y as small as possible consistent with the grains not being broken up under the pressure of the gases in a gun.

In France, with powders of high density (about 1·78 to 1·8) the values of x and y vary from $\frac{2}{3}$ to $\frac{3}{4}$, and if the value $x = y = \frac{2}{3}$, be adopted, we find from the foregoing formula

$$S^1 = \tfrac{7}{9} S, \text{ and } S_1^1 = \tfrac{1}{27} S, \text{ or } S_1^1 = \frac{S^1}{21};$$

so that the advantage of the flat grain is very considerable.

160. The weight of powder burnt at the end of the time t is obtained as follows. Let, as before, V be the velocity of combustion under pressure p, and let $\alpha\,\beta$ and γ be the length of the sides of the grains.

At the end of $d\,t$ these become

$$\alpha - 2\,\mathrm{V}\,d\,t,\quad \beta - 2\,\mathrm{V}\,d\,t,\quad \gamma - 2\,\mathrm{V}\,d\,t,$$

and at the end of the time t

$$\alpha - 2\int_0^t \mathrm{V}\,d\,t,\quad \beta - 2\int_0^t \mathrm{V}\,d\,t,\quad \gamma - 2\int_0^t \mathrm{V}\,d\,t,$$

and the volume of the grain will be

$$v = \left(\alpha - 2\int_0^t \mathrm{V}dt\right)\left(\beta - 2\int_0^t \mathrm{V}dt\right)\left(\gamma - 2\int_0^t \mathrm{V}dt\right),$$

and the volume of powder burnt will be

$$\alpha\,\beta\,\gamma - v.$$

Calling $\psi\,(t)$ the ratio of the powder burnt at the end of the time t, to the total weight of the charge,

$$\psi\,(t) = \frac{\alpha\,\beta\,\gamma - v}{\alpha\,\beta\,\gamma} = 1 - \frac{v}{\alpha\,\beta\,\gamma},$$

and replacing v by its value found above

$$(a)\quad \psi\,(t) = 1 - \left(1 - \frac{2}{\alpha}\int_0^t \mathrm{V}dt\right)\left(1 - \frac{2}{\beta}\int_0^t \mathrm{V}d\,t\right)$$
$$\left(1 - \frac{2}{\gamma}\int_0^t \mathrm{V}d\,t\right).$$

161. In free air V is constant, and if it is denoted by V_0

$$(b)\quad \psi\,(t) = 1 - \left(1 - \frac{2}{\alpha}\,\mathrm{V}_0\,t\right)\left(1 - \frac{2}{\beta}\,\mathrm{V}_0\,t\right)\left(1 - \frac{2}{\gamma}\,\mathrm{V}_0\,t\right).$$

If τ be the total duration of combustion of the grain

$$\alpha - 2\,\mathrm{V}_0\,\tau = 0,$$

whence

$$V_0 = \frac{a}{2\tau},$$

using which in (*b*)

$$\psi(t) = 1 - \left(1 - \frac{t}{\tau}\right)\left(1 - \frac{a}{\beta}\frac{t}{\tau}\right)\left(1 - \frac{a}{\gamma}\frac{t}{\tau}\right),$$

and making

$$x = \frac{a}{\beta}, \quad y = \frac{a}{\gamma},$$

$$\psi(t) = 1 - \left(1 - \frac{t}{\tau}\right)\left(1 - x\frac{t}{\tau}\right)\left(1 - y\frac{t}{\tau}\right),$$

which may be put into the form

$$\psi(t) = (1 + x + y)\frac{t}{\tau}\left(1 - \frac{x + y + xy}{1 + x + y} \cdot \frac{t}{\tau} + \frac{xy}{1 + x + y} \cdot \frac{t^2}{\tau^2}\right);$$

or if we write

$$a = 1 + x + y, \quad \lambda = \frac{x + y + xy}{1 + x + y}, \quad \mu = \frac{xy}{1 + x + y}.$$

$$(c) \quad \psi(t) = a\frac{t}{\tau}\left(1 - \lambda\frac{t}{\tau} + \mu\frac{t^2}{\tau^2}\right).$$

The weight of powder burnt will be $w \,.\, \psi(t)$ and the velocity of emission or

$$\eta = \frac{d \,.\, w\,\psi(t)}{dt},$$

is

$$\eta = w \,.\, \frac{a}{\tau}\left(1 - \frac{2\lambda}{\tau} \,.\, t + \frac{3\mu}{\tau^2}t^2\right).$$

With cubical grains $x = y = 1$, and

$$a = 3, \quad \lambda = 1, \quad \mu = \tfrac{1}{3}.$$

$$\eta = w\frac{3}{\tau}\left(1 - \frac{2}{\tau} \,.\, t + \frac{t^2}{\tau^2}\right) = w\,\frac{3}{\tau}\left(1 - \frac{t}{\tau}\right)^2.$$

With flat grains, if

$$x = y = \tfrac{2}{3},$$

and

$$a = \tfrac{7}{3}, \quad \lambda = \tfrac{16}{21}, \quad \mu = \tfrac{4}{21},$$

$$\eta = w\left(\frac{7}{3\tau} - \frac{32\,t}{9\,\tau^2} + \frac{4\,t^2}{3\,\tau^3}\right).$$

162. The formula (a) (§ 160) may be applied to the combustion in the chamber of a gun, only in this case V is no longer constant, but is a function of the pressure.

M. Sarrau arrived at the conclusion, that the relation might be expressed by the formula

$$V = K p^{a'},$$

and that the value of a' was $\frac{1}{2}$, therefore if V_0 be the velocity at atmospheric pressure $= p_0$

$$V = V_0 \left(\frac{p}{p_0}\right)^a = V_0 \left(\frac{p}{p_0}\right)^{\frac{1}{2}}.$$

If therefore p be taken as the pressure in the chamber, the above formula becomes

$$\psi(t) = 1 - \left(1 - \frac{2\,V_0}{a}\int\left(\frac{p}{p_0}\right)^{a'} d\,t\right)\left(1 - \frac{2\,V_0}{\beta}\int_0^t\left(\frac{p}{p_0}\right)^{a'} d\,t\right)\left(1 - \frac{2\,V^0}{\gamma}\int_0^t\left(\frac{p}{p_0}\right)^{a'} d\,t\right),$$

which, proceeding as before, may be put under the form

$$(a)\quad \psi(t) = \frac{a}{\tau}\int_0^t\left(\frac{p}{p_0}\right)^{a'} d\,t\left\{1 - \frac{\lambda}{\tau}\int_0^t\left(\frac{p}{p_0}\right)^{a'} d\,t + \frac{\mu}{\tau^2}\left(\int_0^t\left(\frac{p}{p_0}\right)^{a'} d\,t\right)^2\right\}.$$

In like manner for any other form of grain, the combustion under pressure may be deduced from the combustion in the open air by substituting for t the definite integral

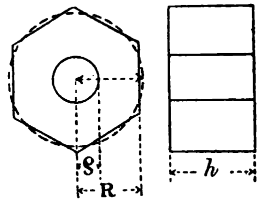
$$\int_0^t\left(\frac{p}{p_0}\right)^{a'} d\,t.$$

163. In the case of prismatic powder with a central hole the calculation is as follows:—

Let ρ be the radius of the central hole;

R, the mean radius of the inscribed and circumscribed circle, which for simplicity is used instead of the real periphery.

h = the height of the prisms.



Then the original volume $= \pi h (R^2 - \rho^2)$, and at the end of the time t it will be

$$v = \pi \{(R - V_0 t)^2 - (\rho + V_0 t)^2\} (h - 2 V_0 t),$$

and the volume of powder burnt will be

$$\pi (R^2 - \rho^2) h - v,$$

and the function

$$\psi (t) = \frac{\pi (R^2 - \rho^2) h - v}{\pi (R^2 - \rho^2) h}.$$

Usually $R - \rho < h$,

and $$2 V_0 \tau = R - \rho, \quad \text{or} \quad V_0 = \frac{R - \rho}{2 \tau},$$

where τ is the total time of burning, and substituting this for v,

$$v = \pi (R^2 - \rho^2) \left(1 - \frac{t}{\tau}\right) \left(h - \frac{R - \rho}{\tau} . t\right)$$

and

$$\psi (t) = 1 - \left(1 - \frac{t}{\tau}\right) \left(1 - \frac{R - \rho}{h} \cdot \frac{t}{\tau}\right);$$

or making

$$x = \frac{R - r}{h},$$

$$\psi (t) = (1 + x) \frac{t}{\tau} - x \frac{t^2}{\tau^2}.$$

164. The weight of powder burnt at the time t will be $w \cdot \psi(t)$ and the velocity of emission

$$\eta = \frac{d \cdot w \psi(t)}{d t}$$

wrong

will be expressed by

$$\eta = w\left(1 + x - \frac{2x}{\tau^2} \cdot t\right).$$

$$\eta = \frac{w}{\tau}\left(1 + x - 2x\frac{t}{\tau}\right)$$

165. It must be borne in mind that the above is on the assumption that the whole surface of the grain is simultaneously ignited, a condition which does not obtain where prismatic grains are closely packed in a cartridge.

166. Reverting now to the equation (7) (§ 148),

$$(x + z)\frac{d^2 x}{d t^2} + \theta\left(\frac{d x}{d t}\right)^2 = \frac{f q}{m}. \tag{7}$$

q is the weight of the powder burnt at the time t, and if w be the total weight of the charge

$$\frac{q}{w} = \psi(t), \quad \text{or,} \quad q = w \psi(t).$$

As is shown above (§ 162) the expression for $\psi(t)$ for powder burnt under pressure in a gun, may be deduced from that obtained from combustion in free air by substituting for t the definite integral $\int_0^t \left(\frac{p}{p_0}\right)^{a'} d t$.

167. In the case of a charge of parallelopipedal grains burning in free air, it has been shown that

$$\psi(t) = a\frac{t}{\tau}\left(1 - \lambda\frac{t}{\tau} + \mu\frac{t^2}{\tau^2}\right),$$

and that with other forms of grain $\psi(t)$ may be developed in terms of increasing powers of t, only the values of the coefficients a, λ, and μ will not be the same.

168. In the integral $\int_0^t \left(\frac{p}{p_0}\right)^{a'}$, p_0 is the atmospheric pressure, and p is the mean pressure at the time $t = P_1$

Now m being the mass of the projectile, ω the area of the bore,

$$P_{,} = \frac{m}{\omega} \cdot \frac{d^2 x}{d t^2};$$

therefore

$$\int_0^t \left(\frac{p}{p_0}\right)^{\alpha'} = \left(\frac{m}{\omega p_0}\right)^{\alpha'} \int_0^t \left(\frac{d^2 x}{d t^2}\right)^{\alpha'} d t,$$

and making use of this instead of t in $\psi\, t$ we get

$$q = w \psi (t) = w \frac{a}{\tau} \left(\frac{m}{\omega p_0}\right)^{\alpha'} \int_0^t \left(\frac{d^2 x}{d t^2}\right)^{\alpha'} d t$$

$$\left\{1 - \frac{\lambda}{\tau}\left(\frac{m}{\omega p_0}\right)^{\alpha'} \int_0^t \left(\frac{d^2 x}{d t^2}\right)^{\alpha'} dt + \frac{\mu}{\tau}\left(\frac{m}{\omega \mu_0}\right)^{2\alpha'} \left(\int_0^t \left(\frac{d^2 x}{d t^2}\right)^{\alpha'} d t\right)^2\right\} \cdot (8)$$

Therefore combining (7) and (8) we get

$$(x + z) \frac{d^2 x}{d t^2} + \theta \left(\frac{d x}{d t}\right)^2 = \frac{f a w}{m \tau} \left(\frac{m}{\omega p_0}\right)^{\alpha'} \int_0^t \left(\frac{d^2 x}{d t^2}\right)^{\alpha'} d t$$

$$\left\{1 - \frac{\lambda}{\tau}\left(\frac{m}{\omega p_0}\right)^{\alpha'} \int_0^t \left(\frac{d^2 x}{d t^2}\right)^{\alpha'} d t + \frac{\mu}{\tau^2}\left(\frac{m}{\omega p_0}\right)^{2\alpha'} \left[\int_0^t \left(\frac{d^2 x}{d t^2}\right)^{\alpha'} d t\right]^2\right\} (9)$$

which is the equation of motion of the projectile.

169. The problem therefore is, to find a function x of t such as to satisfy equation (9), and such that it, as well as its first differential coefficient, become zero when $t = 0$.

M. Sarrau has effected this by the use of certain auxiliary functions, purely numerical and independent of the variable ballistic elements ("éléments variables du tir"). Moreover the general expressions for the velocity and pressure may be put into the form of rapidly converging series, defined by these auxiliary functions, so that by neglecting the terms following the two first, a binomial formula is obtained for the velocity, which for any value of x, the travel of the projectile, gives very approximately the velocity, and by making x equal the total length of the travel, the muzzle velocity is obtained.

As regards the maximum pressure, the series is such that it is sufficiently accurate to retain the first term only.

170. Those who wish to make themselves acquainted with the details of M. Sarrau's analysis, will find it fully gone into in 'Mémorial do l'Artillerie de la Marine,' vol. iv. and

subsequent volumes, and therefore I will not enter into details here, but proceed to the results obtained by M. Sarrau.

Binomial Formula for Velocity.

171. M. Sarrau adopts $\frac{1}{2}$ for the value of a', the index of the term $\frac{p}{p_0}$, on which the velocity of burning depends, and the resulting formula for the velocity of the projectile is

$$V = \left(\frac{f a w}{\tau m}\right)^{\frac{1}{2}} \left(\frac{m z}{\omega p_0}\right)^{\frac{1}{4}} \left\{\phi_0\left(\frac{l}{z}\right) - \frac{\lambda}{\tau}\left(\frac{m z}{\omega p_0}\right)^{\frac{1}{2}} \phi_1\left(\frac{l}{z}\right)\right\}, \quad (8)$$

where l is the travel of the projectile, and z the reduced length of the initial void as defined above (§ 118).

172. M. Sarrau shows that where $\frac{l}{z}$ is large

$$\phi_0\left(\frac{l}{z}\right) = A_0\left(\frac{l}{z}\right)^{\frac{1}{4}}, \quad \text{and} \quad \phi_1\left(\frac{l}{z}\right) = A_1\left(\frac{l}{z}\right)^{\frac{3}{4}},$$

where A_0 and A_1 are numerical coefficients.

Let (8) (§ 171) be written

$$V = \left(\frac{f a w}{\tau m}\right)^{\frac{1}{2}} \left(\frac{m z}{\omega p_0}\right)^{\frac{1}{4}} \phi_0\left(\frac{l}{z}\right) \left\{1 - \frac{\lambda}{\tau}\left(\frac{m z}{\omega p_0}\right)^{\frac{1}{2}} \cdot \frac{\phi_1\left(\frac{l}{z}\right)}{\phi_0\left(\frac{l}{z}\right)}\right\}. \quad (9)$$

Now under ordinary conditions, the ratio $\frac{l}{z}$ is large whilst the second term of the quantity within the brackets is in general a small fraction of unity. Consequently there may be substituted for the ratio $\frac{\phi_1}{\phi_0}$, the value to which it tends for increasing values of $\frac{l}{z}$, or $\frac{A_1}{A_0}\left(\frac{l}{z}\right)^{\frac{1}{2}}$.

Besides, $\frac{l}{z}$ usually varying between relatively narrow limits, it is possible to replace the unknown function ϕ_0 by a function $Q\left(\frac{l}{z}\right)^{\gamma}$, provided the coefficient Q and the index γ be suitably chosen.

Making these substitutions (9) becomes

$$V = Q\left(\frac{fa}{\tau}\right)^{\frac{1}{2}}\left(\frac{w}{m}\right)^{\frac{1}{4}}\left(\frac{w}{\omega p_0}\right)^{\frac{1}{4}} z^{\frac{1}{4}-\gamma}\, l^{\gamma}\left\{1 - \frac{A_1}{A_0}\frac{\lambda}{\tau}\left(\frac{m\,l}{\omega p_0}\right)^{\frac{1}{2}}\right\}. \quad (10)$$

Replacing z by its value (§ 118)

$$\frac{\frac{w}{\Delta} - \frac{w}{\delta}}{1000\,\omega}{}^{*},$$

and observing that $m = \frac{W}{g}$, $\omega = \frac{\pi c^2}{4}$ where c is the calibre of the gun, and W the weight of the projectile, we get

$$V = A\left(\frac{fa}{\tau}\right)^{\frac{1}{2}}\left(\frac{w}{W}\right)^{\frac{1}{4}}\left(\frac{w}{c^2}\right)^{\frac{1}{2}-\gamma}\left(\frac{1}{\Delta} - \frac{1}{\delta}\right)^{\frac{1}{4}-\gamma} l^{\gamma}\left\{1 - B\,\frac{\lambda}{\tau}\frac{(W\,l)^{\frac{1}{2}}}{c}\right\}. \quad (11)$$

A and B being numerical coefficients, viz.

$$A = \frac{Q}{1000}\left(\frac{g}{p_0}\right)^{\frac{1}{4}}\left(\frac{4}{\pi}\right)^{\frac{1}{2}-\gamma}, \text{ and } B = \frac{A_1}{A^1_0}\left(\frac{4}{\pi\, g\, p_0}\right)^{\frac{1}{2}}.$$

Now $\frac{\Delta}{\delta}$ differs very little from $\frac{1}{2}$, for which value $\frac{\Delta}{\delta}\left(1 - \frac{\Delta}{\delta}\right)$ attains its maximum $= \frac{1}{4}$; therefore we may make $\frac{\Delta}{\delta}\left(1 - \frac{\Delta}{\delta}\right) = \frac{1}{4}$ and $\left(\frac{1}{\Delta} - \frac{1}{\delta}\right) = \frac{\delta}{4\,\Delta^2}$,

introducing which into (11) we get

$$V = A\left(\frac{fa}{\tau}\right)^{\frac{1}{2}}\frac{w^{\frac{3}{4}-\gamma}\,\Delta^{2\gamma-\frac{1}{2}}\, l^{\gamma}}{W^{\frac{1}{4}}\, c^{1-2\gamma}\,\delta^{\gamma-\frac{1}{4}}}\left\{1 - B\,\frac{\lambda}{\tau}\frac{(W\,l)^{\frac{1}{2}}}{c}\right\}. \quad (12)$$

It has been shown by the experiments of Col. Desbordes ('Mémorial de l'Artillerie de la Marine,' vol. vii. p. 441), that the velocity varies as the $\frac{5}{8}$th power of the weight of the charge when the gravimetric density ("densité de chargement") is constant, and as the $\frac{1}{4}$th power of the gravimetric density when the weight of charge is constant.

Consequently we must make $\gamma = \frac{3}{8}$.

* The division 1000 is introduced to bring the unity of volume of z into cubic decimetres.

Introducing this value, and including in the constant A the factor $\delta^{\frac{1}{8}}$ which is very nearly constant, the formula finally takes the form of

$$V = A\left(\frac{f\,a}{\tau}\right)^{\frac{1}{2}} (w\,l)^{\frac{3}{8}} \left(\frac{\Delta}{W\,c}\right)^{\frac{1}{4}} \left\{1 - B\,\frac{\lambda}{\tau}\,\frac{(W\,l)^{\frac{1}{2}}}{c}\right\}.$$

173. The terms $\left(\frac{f\,a}{\tau}\right)^{\frac{1}{2}}$ and $\frac{\lambda}{\tau}$ depend solely on the powder employed, and they are termed by M. Sarrau the "Characteristics" of the powder, and are denoted by α and β respectively, so that

$$V = A\,\alpha\,(w\,l)^{\frac{3}{8}} \left(\frac{\Delta}{W\,c}\right)^{\frac{1}{4}} \left\{1 - B\,\beta\,\frac{(W\,l)^{\frac{1}{2}}}{c}\right\} \qquad (13)$$

which is M. Sarrau's binomial formula for the velocity.

Formula for Maximum Pressure on Base of Projectile.

174. In the integration of the general formula (9) § 168, M. Sarrau changes the variables by making $\frac{x}{z} = y$, and writing K for $\frac{f\,a\,w}{m\,z^2\,\tau}\left(\frac{m\,z}{\omega\,p_0}\right)^{a'}$, β for $\frac{1}{3 - 2\,a'}$, ξ for $K^{\beta}\,t$, and ϵ for $\frac{1}{-}\left(\frac{m\,z}{\omega\,p_0}\right)^{a'} K^{\frac{2\,a' - 1}{3 - 2\,a'}}$, he obtains the following formula for the maximum pressure—

$$P = \frac{m\,z}{\omega}\,K^{2\beta}\left(\frac{d^2\,y_0}{d\,\xi^2} + \epsilon\,\frac{d^2\,y_1}{d\,\xi^2} + \&c.\right); \qquad (14)$$

and since ϵ is always very small it is sufficient to take the first term of the series. Therefore

$$P = \frac{m\,z}{\omega}\,K^{2\beta}\left(\frac{d^2\,y_0}{d\,\xi^2}\right)_{m}.$$

the subscript m denoting the maximum value of the function $\frac{d^2 y_1}{d\xi^2}$.

The function y_0 is a purely numerical function of ξ, and of $\frac{d^2 y_0}{d\xi^2}$, so that the maximum value of this function is a number which may be denoted by N.

Replacing K and β by their values, and writing $a = \frac{1}{2}$

$$P = N \frac{f a w}{\omega \tau} \cdot \left(\frac{m}{z \omega p_0}\right)^{\frac{1}{2}};$$

and since

$$m = \frac{W}{g}, \quad \omega = \frac{\pi c^2}{4}, \quad z = \frac{w}{1000 \omega}\left(\frac{1}{\Delta} - \frac{1}{\delta}\right)$$

or

$$z = \frac{w}{1000 \omega} \cdot \frac{\delta}{4 \Delta^2} \text{ as before.}$$

Writing

$$K = \frac{8 N \times 10^{\frac{3}{2}}}{\pi (g p_0 \delta)^{\frac{1}{2}}},$$

and since $\delta^{\frac{1}{2}}$ is very nearly constant, including it in K we get finally

$$P = K \frac{f a}{\tau} \Delta \frac{(w l)^{\frac{1}{2}}}{c^2}, \text{ and since } \left(\frac{f a}{\tau}\right)^{\frac{1}{2}} = a,$$

$$P = K a^2 \Delta \frac{(w W)^{\frac{1}{2}}}{c^2} \tag{15}$$

which is M. Sarrau's formula for the maximum pressure on the base of the projectile.

Maximum Pressure on Breech of Gun.

175. The above formula deduced from the acceleration of the projectile, gives the maximum pressure on its base.

From it may be deduced the maximum pressure on the breech as follows ;—

Let M, m, and μ be the masses of the gun with its carriage, the projectile, and the charge respectively, and v_1, v the velocities of the two former,

As regards the velocity of the charge, it is evidently less than that of the projectile, and it may be denoted by $\theta^1 v$ where θ^1 is a coefficient less than unity.

Consequently $m\,v - \mathrm{M}\,v_1 + \theta^1\,m\,v = 0$, and differentiating with respect to the time,

$$\frac{d\,v_1}{d\,t} = \frac{m + \theta^1\,\mu}{\mathrm{M}}\,\frac{d\,v}{d\,t},$$

whence

$$\frac{\mathrm{M}}{\omega}\cdot\frac{d\,v^1}{d\,t} = \frac{m}{\omega}\,\frac{d\,v}{d\,t}\left(1 + \theta^1\,\frac{\mu}{m}\right)$$

and denoting by P_0 and P the pressures per unit of surface on the breech and base of projectile respectively,

$$\mathrm{P}_0 = \mathrm{P}\left(1 + \theta^1\,\frac{\mu}{m}\right) = \mathrm{P}\left(1 + \theta^1\,\frac{w}{\mathrm{W}}\right) = \mathrm{P}\left(1 + \frac{w}{2\,\mathrm{W}}\right)$$

if θ^1 be taken $= \frac{1}{2}$.

176. The value of $\theta^1 = \frac{1}{2}$ is based upon two hypotheses of General Piobert:—

(*a*) That the density and temperature of the products are uniform throughout the space between the breech and the projectile.

(*b*) That if the whole mass be divided into infinitely thin slices at right angles to the axis, the velocity of each slice is proportionate to its distance from the breech.

Neither of these hypotheses is exact. Moreover, in making $\mathrm{P} = \frac{m}{\omega}\cdot\frac{d\,v}{d\,t}$ its value is certainly somewhat too small as there are certain small passive resistances which have been neglected, such as friction, &c. There is probably also a certain amount of vis viva lost, during the process of combustion, by the gases striking against the walls of the chamber. Having regard to these and other considerations, M. Sarrau concludes that the true value of θ^1 is $\frac{3}{3}$, and the above formula becomes $\mathrm{P}_0 = \mathrm{P}\left(1 + \frac{3}{3}\,\frac{w}{\mathrm{W}}\right)$, which appears to agree very well with the results of

nor less than unity

experience. Consequently, to obtain the maximum pressure on the breech, the pressure on the base of the projectile must be multiplied by the factor $1 + \frac{3}{2}\frac{w}{W}$ and we get

$$P_0 = K\left(1 + \frac{3}{2}\frac{w}{W}\right) a^2 \Delta \frac{(w\,W)^{\frac{1}{2}}}{c^2} \qquad (16)$$

for the breech pressure.

177. For practical use, a monomial formula is more convenient, and this may be obtained by observing that any increasing function may, within certain limits, be considered proportional to some positive power of its variable, therefore making

$$1 + \frac{3}{2}\frac{w}{W} = K_1 \left(\frac{w}{W}\right)^{\gamma}$$

we get

$$P_0 = K_1 K \left(\frac{w}{W}\right)^{\gamma} \frac{f\,a}{\tau} \Delta \frac{(w\,W)^{\frac{1}{2}}}{c^2};$$

or making $K_1 K = K_0$ and $\gamma = \frac{1}{4}$, which has been found sufficiently to agree with practical results, we get finally

$$P_0 = K_0 \left(\frac{w}{W}\right)^{\frac{1}{4}} a^2 \frac{\Delta\,(w\,W)^{\frac{1}{2}}}{c^2} \qquad (17)$$

which is M. Sarrau's formula for the maximum pressure on the breech.

Position of Projectile corresponding to Maximum Pressure.

178. In (§ 174) the symbol ϵ was introduced

$$= \frac{1}{\tau}\left(\frac{m\,z}{\omega\,p_0}\right)^{a'} K^{\frac{2a'-1}{3-2a'}}.$$

Now it is shown by M. Sarrau ('Mémorial de l'Artillerie de la Marine,' vol. iv. p. 189) that ϵ is generally very small, and

that the function $y = \frac{x}{z}$ may be developed in ascending powers of ϵ in the form $y = y_0 + \epsilon y_0 + \epsilon^2 y_2$, &c., &c., where y_0, y_1, y_2, &c., are unknown functions of ζ $(= K^\beta t)$ and that these functions are defined by the relations

$$(y_0 + 1)\frac{d^2 y_0}{d\zeta^2} + \theta\left(\frac{dy}{d\zeta}\right)^2 = X_0 \tag{18}$$

$$(y_0 + 1)\frac{d^2 y_1}{d\zeta^2} + 2\theta\frac{dy_0}{d\zeta}\cdot\frac{dy_1}{d\zeta} + \frac{d^2 y_0}{d\zeta^2}y_1 = X_1 - \lambda X_0^2, \tag{19}$$

&c. &c. &c.

and

$$X_0 = \int_0^\zeta \left(\frac{d^2 y_0}{d\zeta^2}\right)^a d\zeta$$

$$X_1 = a\int_0^\zeta \frac{d^2 y_1}{d\zeta^2}\left(\frac{d^2 y_0}{d\zeta^2}\right)^{a-1} d\zeta.$$

&c. &c. &c.

179. Making use of these relations M. Sarrau has calculated the value N of the maximum of the function $\frac{d^2 y_0}{d\zeta^2}$ (§ 174) and deduced therefrom the value x_m of the space passed through at the time of maximum pressure, in the following manner.

180. Let, as before (§ 174), N be the maximum value of $\left(\frac{d^2 y_0}{d\zeta^2}\right)$.

Let $n = \frac{c^1}{c}$ the ratio of the specific heats at constant pressure and constant volume, which M. Sarrau takes at 1·40.*

Then

$$\theta = \frac{n-1}{2} = \tfrac{1}{5},$$

* By Noble and Abel this is = 1·322.

and if $a = \frac{1}{2}$

$$X_0 = \int_0^{\zeta} \left(\frac{d^2 y_0}{d\zeta^2}\right)^{\frac{1}{2}} d\zeta$$

and equation (19) becomes

$$(y_0 + 1)\frac{d^2 y}{d\zeta^2} + \tfrac{1}{5}\left(\frac{d y_0}{d\zeta}\right)^2 = \int_0^{\zeta}\left(\frac{d^2 y}{d^2 \zeta}\right)^{\frac{1}{2}} d\zeta. \qquad (20)$$

181. Although this equation is not directly integrable, it is possible, as in manner following, to calculate the values of the function y_0 and its successive differential coefficients for progressively increasing values of the variables.

For this purpose assume

$$\frac{d^2 y_0}{d\zeta^2} = a\zeta^m + b\zeta^n + c\zeta^p + \&c.$$

and integrating

$$\frac{d y_0}{d\zeta} = \frac{a}{m+1}\zeta^{m+1} + \frac{b}{n+1}\zeta^{n+1} + \frac{c}{p+1}\zeta^{p+1} + \&c.$$

and integrating again

$$y_0 = \frac{a}{(m+1)(m+2)}\zeta^{m+2} + \frac{b}{(n+1)(n+2)}\zeta^{n+2} + \&c.$$

Introducing these values into (20), the coefficients and exponents are determined, and series are obtained which are very convergent, when $\zeta < 1$ or $K^{\beta} t < 1$, that is to say, for $K^{\frac{1}{3-2a'}} < 1$, or taking $a' = \frac{1}{2}$ this comes to $K^{\frac{1}{2}} t < 1$ or

$$\log^{-1} 1{\cdot}98098\, t \text{ or } 95{\cdot}71\, t < 1,$$

that is to say, for values of t less than $\frac{1}{95{\cdot}71}$ of a second.

For values of $\zeta > 1$ recourse must be had to developments by Taylor's theorem.

182. Having calculated by means of the preceding series

$$y_0,\ \frac{d y_0}{d\zeta},\ \frac{d^2 y}{d\zeta^2},$$

for a value ζ_1 of ζ less than unity, we must calculate successively by means of these three values, the differential

coefficients of y_0, of higher orders for $\zeta = \zeta_1$, making use of the series of equations obtained by successive differentiation of the equation (20).

Making then

$$\frac{d^2 y_0}{d\zeta^2} = \left[\frac{d^2 y_0}{d\zeta^2}\right]_{\zeta_1} + \left[\frac{d^3 y_0}{d\zeta^3}\right]_{\zeta_1}(\zeta-\zeta_1) + \left[\frac{d^4 y_0}{d\zeta^4}\right]_{\zeta_1}\frac{(\zeta-\zeta_1)^2}{2} + \&c., \quad (21)$$

and integrating twice between the limits ζ and ζ_1,

$$\frac{d y_0}{d\zeta} = \left[\frac{d y_0}{d\zeta}\right]_{\zeta_1} + \left[\frac{d^2 y_0}{d\zeta^2}\right]_{\zeta_1}(\zeta-\zeta_1) + \left[\frac{d^3 y_0}{d\zeta^3}\right]_{\zeta_1}\frac{(\zeta-\zeta_1)^2}{2} + \&c.;$$

and

$$y_0 = [y_0]_{\zeta_1} + \left[\frac{d y_0}{d\zeta}\right]_{\zeta_1}(\zeta-\zeta_1) + \left[\frac{d^2 y_0}{d\zeta^2}\right]_{\zeta}\frac{(\zeta-\zeta_1)^2}{2},$$

we may calculate $y_0, \frac{d y_0}{d\zeta}, \frac{d^2 y}{d\zeta^2}$ for a value ζ_2 of ζ greater than ζ_1.

We may thus obtain by means of equations deduced by differentiation of equation (20), the differential coefficients of y_0 of the higher degrees for $\zeta = \zeta_2$, and by means of a formula analogous to (21) pass to a value ζ_3 greater than ζ_2, and so on.

183. In this way the following table was obtained.

ζ	y_0	$\frac{dy_0}{d\zeta}$	$\frac{d^2 y_0}{d\zeta^2}$
1·00	0·021	0·082	0·242
2·00	0·303	0·557	0·666
2·25	0·464	0·729	0·704
2·50	0·668	0·906	0·709
2·75	0·917	1·082	0·690
3·00	1·208	1·250	0·657
5·00	4·809	2·254	0.372

From this it appears that the maximum value of

$$\frac{d^2 y_0}{d\zeta^2} = 0{\cdot}709$$

when the corresponding value of y_0 is 0·668, but $y = \frac{x}{z}$, whence $x = zy = zy_0 + z\epsilon y_1 + \&c.$, and approximately (neglecting the latter terms of the development as ϵ is very small)

$$x = zy_0 = {\cdot}668\, z,$$

but

$$z = \frac{w}{1000\,\omega}\left(\frac{1}{\Delta} - \frac{1}{\delta}\right),$$

therefore

$$x_m = 0\cdot 668\,\frac{w}{1000\,\omega}\cdot\left(\frac{1}{\Delta} - \frac{1}{\delta}\right)$$

$$= \frac{\cdot 000668\,w}{\omega}\cdot\left(\frac{1}{\Delta} - \frac{1}{\delta}\right).$$

184. Since ω is expressed in square metres, the distance x_m is given in metres;
or since

$$\omega = \frac{\pi\,c^2}{4}$$

$$x_m = \frac{\cdot 00084\,w}{c^2}\left(\frac{1}{\Delta} - \frac{1}{\delta}\right);$$

and if c be in decimetres and x_m in metres

$$x_m = \frac{\cdot 0845\,w}{c^2}\left(\frac{1}{\Delta} - \frac{1}{\delta}\right);$$

or if c and x_m be both in decimetres,

$$x_m' = \frac{\cdot 845\,w}{c^2}\left(\frac{1}{\Delta} - \frac{1}{\delta}\right). \qquad (22)$$

Monomial Formula for Velocity.

185. In investigating this formula, M. Sarrau proceeds on the assumption, that the velocity in a gun with a given kind of powder, is a function of the five following variables:—

w = weight of charge;
Δ = gravimetric density (densité de chargement);
l = length of travel of projectile;
c = calibre;
W = weight of projectile;

and that this function is proportional, within certain limits, to some power of each variable.

186. The velocity may therefore be represented by

$$V = H \cdot \frac{w^{\alpha} \Delta^{\beta} l^{\gamma} c^{\epsilon}}{W^{\eta}},$$

the exponents of which must be determined experimentally, and H being a constant depending on the kind of powder used.

187. The Commission of Gavre long ago determined the values of α, β, and η, with an approximation quite sufficient for practice to be

$$\alpha = \tfrac{3}{8}, \quad \beta = \tfrac{1}{4}, \quad \eta = \tfrac{4}{10}.$$

From certain experiments made with the powder $W\frac{13}{16}$ in a 24 cm. (9·45 inch) gun, the value of γ was found to be $\frac{1}{8}$, but it increases as the powder becomes slower, relatively to the gun, as for instance when the same $W\frac{13}{16}$ powder was used in a 10 cm. (4 inch) gun the value of γ rose to $\frac{1}{4}$.

The value of ϵ is still unknown. It follows therefore that the empirical formulæ hitherto in use, whilst they may give with sufficient approximation the different velocities in one and the same gun, give no indication of the results of firing the same powder in guns of different calibres.

188. M. Sarrau, in his various papers (in the 'Mémorial de l'Artillerie de la Marine,' vols. ii., iv., v., and vi.) has established three relations between these exponents, so that it is sufficient to know two in order to determine the others. The formula thus arrived at is less exact than the binomial formula already given, but the approximation is nevertheless satisfactory. It has, moreover, the advantage of including only one constant. Consequently, within the limits of its application, it suffices to know the velocity in any one gun, in order to determine the velocities in other guns with the same powder.

189. The formula is established as follows:—

The theory of the effect of powder in a gun shows, that a

monomial expression representing approximately the initial velocity is necessarily of the form

$$V = B\left(\frac{f\,a\,w}{\tau\,m}\right)^{\frac{1}{2}}\left(\frac{m\,z}{\omega\,p_0}\right)^{1-\frac{2\gamma}{4}}\left(\frac{\tau}{\lambda}\right)^{\gamma'}\left(\frac{l}{z}\right)^{\gamma} \qquad (23)$$

where

$$z = \frac{w}{\omega}\left(\frac{1}{\Delta} - \frac{1}{\delta}\right),$$

and since $m = \dfrac{W}{g}$, $\omega = \dfrac{\pi\,c^2}{4}$, introducing these values and that of z

$$V = M\left(\frac{f\,a}{\tau}\right)^{\frac{1}{2}}\left(\frac{\tau}{\lambda}\right)^{\gamma'}\frac{w^{\alpha}\,l^{\gamma}\,c^{\epsilon}}{W^{\eta}}\left(\frac{1}{\Delta} - \frac{1}{\delta}\right)^{-\frac{\beta}{2}}. \qquad (24)$$

M being a constant independent of all the ballistic elements, and the exponents having the following values :—

$$\left.\begin{array}{ll} \alpha = \frac{3}{4} - \dfrac{2\gamma + \gamma'}{2}, & \beta = 2\gamma + \gamma' - \frac{1}{2} \\ \epsilon = 2\gamma + 2\gamma' - 1, & \eta = \frac{1}{4} + \dfrac{\gamma'}{2} \end{array}\right\} \qquad (25)$$

190. The formula (24) is reduced to a more simple form by the consideration that the value of a function remains sensibly constant in the vicinity of its maximum.

Now, in ordinary conditions of practice, the ratio $\dfrac{\Delta}{\delta}$ differs very little from $\frac{1}{2}$, which value gives to the function $\dfrac{\Delta}{\delta}\left(1 - \dfrac{\Delta}{\delta}\right)$ is maximum value of $\frac{1}{4}$. Consequently, we get approximately

$$\frac{\Delta}{\delta}\left(1 - \frac{\Delta}{\delta}\right) = \frac{1}{4}$$

and

$$\frac{1}{\Delta} - \frac{1}{\delta} = \frac{1}{4}\,\frac{\delta}{\Delta^2};$$

and having regard to this approximate relation, we may substitute for $\left(\frac{1}{\Delta} - \frac{1}{\delta}\right)^{-\frac{\beta}{2}}$ the product of $\Delta^{\beta}\,\delta^{-\frac{\beta}{2}}$ by a numerical factor which may be included in the constant M.

And making

$$\mathrm{H} = \mathrm{M}\,\delta^{-\frac{\beta}{2}}\left(\frac{f\,a}{\tau}\right)^{\frac{1}{2}}\left(\frac{\tau}{\lambda}\right)^{\gamma'}$$

we get the formula of (§ 186) or

$$\mathrm{V} = \mathrm{H}\,\frac{w^{a}\,\Delta^{\beta}\,l^{\gamma}\,c^{\epsilon}}{\mathrm{W}^{\eta}}, \tag{26}$$

H being a constant depending on the powder, and the exponents having the values given in (25).

These exponents are six in number and are functions of γ and γ'.

191. If, in accordance with the experiments of Gavre, we make $a = \frac{3}{8}$, we get from (25)

$$2\,\gamma + \gamma' = \tfrac{3}{4}.$$

Consequently $\beta = 2\,\gamma + \gamma' - \frac{1}{2} = \frac{1}{4}$, which agrees with the empirical determination by the Commission de Gavre.

Moreover, the relation $2\,\gamma + \gamma' - \frac{3}{4}$, gives

$$\gamma' = \tfrac{3}{4} - 2\,\gamma, \quad \epsilon = \tfrac{1}{2} - 2\,\gamma, \quad \text{and } \eta = \tfrac{5}{8} - \gamma\,;$$

consequently the formula (26) becomes

$$\mathrm{V} = \mathrm{H}\,\frac{w^{\frac{3}{8}}\,\Delta^{\frac{1}{4}}\,l^{\gamma}\,c^{\frac{1}{2}-2\gamma}}{\mathrm{W}^{\frac{5}{8}-\gamma}},$$

and it only remains to find γ to determine the formula completely.

192. It has already been stated that the value of γ increases as the powder becomes slower. The values $\frac{1}{8}$ and $\frac{1}{4}$ were

found, the first from a very quick powder, the second from a very slow powder (i. e. relatively to the guns).

The mean value $\gamma = \frac{3}{16}$ may therefore be considered as approximately true for usual conditions of fire, and admitting this value, the formula becomes

$$V = H \cdot \frac{w^{\frac{3}{8}} \Delta^{\frac{1}{4}} l^{\frac{3}{16}} c^{\frac{1}{8}}}{W^{\frac{7}{16}}} \tag{27}$$

and the value of the constant H, is

$$H = M \delta^{-\frac{1}{8}} \left(\frac{fa}{\tau}\right)^{\frac{1}{2}} \left(\frac{\tau}{\lambda}\right)^{\frac{3}{8}}. \tag{28}$$

193. Under ordinary conditions δ varies very little, so that $\delta^{-\frac{1}{8}}$ may be considered as constant, and, reduced to its mean value, may be included in H, so that

$$H = M \left(\frac{fa}{\tau}\right)^{\frac{1}{2}} \left(\frac{\tau}{\lambda}\right)^{\frac{3}{8}}. \tag{29}$$

194. If γ were taken $= \frac{1}{4}$ another formula, corresponding to a very slow powder, would be obtained, in which c does not appear, viz. —

$$V = H . \frac{w^{\frac{1}{8}} \Delta^{\frac{1}{4}} l^{\frac{1}{4}}}{W^{\frac{3}{8}}}. \tag{30}$$

In this case $\gamma' = \frac{1}{4}$ and

$$H = M \delta^{-\frac{1}{8}} \left(\frac{fa}{\tau}\right)^{\frac{1}{2}} \left(\frac{\tau}{\lambda}\right)^{\frac{1}{4}}, \tag{31}$$

and finally including $\delta^{-\frac{1}{8}}$ in M

$$H = M \left(\frac{fa}{\tau}\right)^{\frac{1}{2}} \left(\frac{\tau}{\lambda}\right)^{\frac{1}{4}}. \tag{32}$$

195. In general, formula (27) is applicable, and it has been verified by M. Sarrau, by the actual results of firing with different powders, and in different guns, and under differing conditions of firing. To effect this verification it is necessery

to know the value of H for each powder, and this is obtained by measuring the velocity obtained from any one gun under determined conditions of firing.

196. In the verifications made by M. Sarrau, he adopted for the condition of fire, that of the "regulation proof for reception of powder"; taking as the corresponding normal velocity, the mean of the two limits of velocity, within which the velocity of the lot of powder should be comprised in order that the powder should pass the proof.

He gives a table of the results of firing 81 rounds with different powders, and very different conditions of firing, such as gravimetric density, proportionate weight of charge and projectile, &c., &c., and with guns varying from 75 mm. (3 inches) to 320 mm. (12⅝ inches), and the mean difference between the calculated and the measured velocities was only 3 metres per second, in velocities varying from 300 metres to 600 metres per second.

It must, however, be borne in mind that the allowed margin ("tolérance") on the proof for reception is 8 to 9 metres per second.

The verification, therefore, made by M. Sarrau is eminently satisfactory.

Theoretical Maximum of Velocity.

197. The relation between the velocity and the other ballistic elements has already been shown to be (§ 172)

$$V = A\left(\frac{f\,a}{\tau}\right)^{\frac{1}{2}}\left(w\,l\right)^{\frac{3}{8}}\left(\frac{\Delta}{W\,c}\right)^{\frac{1}{4}}\left\{1 - B\,\frac{\lambda}{\tau}\,\frac{(W\,l)^{\frac{1}{2}}}{c}\right\}.$$

The form of this expression is such, that by varying τ_1 the function passes through a maximum.

Strictly however, this could only be the case if τ were $= 0$,

that is to say if the combustion of the charge were instantaneous.

If, therefore, the above expression gives a maximum for a finite value of τ, it is because the formula is only approximate. It was obtained by neglecting all the terms of a converging series after the first two. The value of τ corresponding to those first two terms has, however, an important signification. It is a limit below which the variation of τ has only an insensible influence on the velocity, and which it is ~~there~~ disadvantageous to exceed, because, whilst the velocity increases very slightly, the maximum pressure increases rapidly in the inverse ratio of the time of combustion.

Consequently, the consideration of this particular value of τ, called by M. Sarrau "the duration of the maximum" ("durée du maximum") is of great importance in the present question.

198. Equating to zero the differential coefficient of V with respect to τ, obtained from equation (13), and denoting by τ_1 the value of τ corresponding to the maximum of V, we get

$$\tau_1 = 3\,B\,\frac{\lambda\,(W\,l)^{\frac{1}{2}}}{c}. \qquad (33)$$

Here it may be observed that for a determinate form of grain, λ is constant and the value of τ_1 depends only on the calibre, the weight, and the travel of the projectile, and is independent of the weight of charge and gravimetric density.

199. When it is said, as it so often is, that a powder is "slow" or "quick," this expression does not really denote any quality in the powder itself. It depends chiefly on the conditions under which it is used. In fact, in a given gun, the powder is "slow," when the duration of the combustion of the grain is notably superior to that duration which, in that particular gun, corresponds to the theoretic maximum of velocity, and *vice versâ*. Moreover, two powders, fired in different guns, should be considered of the same vivacity, when their durations of combustion are

proportional to the "durations of the maximum," relatively to the two guns used.

200. Let, then, the ratio of the "duration of the maximum" relating to a given gun, to the duration of combustion of a powder in that gun be called the "Modulus of Vivacity," or simply the "Modulus," and be denoted by x; then $x = \frac{\tau_1}{\tau}$.

From this point of view, M. Sarrau adopts the following scale of classification of powders:—

Value of modulus x.	Classification of the powder.
1·0	Very quick
0·9	Quick
0·8	Mean
0·7	Slow
0·6	Very slow

201. Accordingly, from (33), since $x = \frac{\tau_1}{\tau}$,

$$x = 3\,\mathrm{B}\,\frac{\lambda}{\tau}\,\frac{(\mathrm{W}\,l)^{\frac{1}{4}}}{c} \tag{34}$$

or

$$x = 3\,\mathrm{B}\,\beta\,\frac{(\mathrm{W}\,l)^{\frac{1}{4}}}{c} \tag{35}$$

$$\text{since } \beta = \frac{\lambda}{\tau}.$$

Formula for Initial Velocity as a Function of the Modulus.

201. Introducing x in the place of τ in formula (13) for the velocity, a new expression is obtained which will be found of great use. Since $\tau_1 = 3\,\mathrm{B}\frac{\lambda\,(\mathrm{W}l)^{\frac{1}{4}}}{c}$ (§ 198) (13) may be written thus:—

$$\mathrm{V} = \mathrm{A}\left(\frac{f\,a}{\tau_1}\right)^{\frac{1}{2}}(w\,l)^{\frac{3}{8}}\left(\frac{\Delta}{\mathrm{W}\,c}\right)^{\frac{1}{4}}\left(\frac{\tau_1}{\tau}\right)^{\frac{1}{2}}\left\{1 - \tfrac{1}{3}\,\frac{\tau_1}{\tau}\right\}$$

which making $x = \frac{\tau_1}{\tau}$ gives

$$V = \tfrac{1}{3} A \left(\frac{f a}{\tau_1}\right)^{\frac{1}{2}} (w\, l)^{\frac{3}{8}} \left(\frac{\Delta}{W c}\right)^{\frac{1}{4}} x^{\frac{1}{2}} (3 - x)$$

and if τ_1 be replaced by its value and we take

$$f(x) = \tfrac{1}{2} x^{\frac{1}{2}} (3 - x) \tag{36}$$

the formula for the velocity becomes

$$V = \tfrac{2}{3} A (3 B)^{-\frac{1}{2}} \left(\frac{f a}{\lambda}\right)^{\frac{1}{2}} \frac{w^{\frac{3}{8}} \Delta^{\frac{1}{4}} c^{\frac{1}{4}} l^{\frac{1}{8}}}{W^{\frac{1}{2}}} \cdot f(x). \tag{37}$$

Maximum Pressure on Base of Projectile as a Function of the "Modulus."

202. The maximum pressure on the base of the projectile as a function of the modulus is thus obtained :—

Expression (15) above may be written

$$P = K \cdot \frac{f a}{\tau_1} \cdot \frac{\Delta (W w) \frac{1}{2}}{c^2} \cdot \frac{\tau_1}{\tau};$$

and making $\frac{\tau_1}{\tau} = x$, and replacing τ_1 in the denominator by its value $3 B \frac{\lambda (W l)^{\frac{1}{2}}}{c}$ we get

$$P = K (3 B)^{-1} \frac{f a}{\lambda} \frac{\Delta w^{\frac{1}{2}}}{c\, l^{\frac{1}{2}}} \cdot x. \tag{38}$$

203. The maximum pressure on the breech is deduced from this by writing P_0 for P and replacing K by $K_0 \left(\frac{w}{W}\right)^{\frac{1}{4}}$; therefore

$$P_0 = K_0 (3 B)^{-1} \left(\frac{f a}{\lambda}\right) \left(\frac{w}{W}\right)^{\frac{1}{4}} \frac{\Delta w^{\frac{1}{2}}}{c\, l^{\frac{1}{2}}} \cdot x. \tag{39}$$

204. From the above may be obtained a new demonstration of the monomial formula, which serves within certain limits, to give the initial velocity.

As has already been remarked, within a certain limited value of the variables, a function $f(x)$ is nearly proportional to a properly chosen power of the variables.

205. In fact, in order that within certain limits of x, the function $f(x)$ shall be sensibly equal to an expression of the form $N x^n$, it suffices to determine N and n, so that for the value x, the two functions and their first differential coefficients should be equal, or

$$f(x) = N x^n$$

and

$$f^1(x) = n N x^{n-1},$$

from which

$$n = x \frac{f^1(x)}{f(x)}.$$

If, for instance, the function $f(x)$ be that which determines the relations (36) or $f(x) = \frac{1}{2} x^{\frac{1}{2}} (3 - x)$ we find

$$n = \frac{3}{2} \cdot \frac{1 - x}{3 - x}. \tag{40}$$

Thus the expression (37) for the velocity may be put under the form

$$V = \frac{2}{3} A (3 B)^{-\frac{1}{2}} N \left(\frac{f a}{\lambda}\right)^{\frac{1}{2}} \frac{w^{\frac{3}{8}} \Delta^{\frac{1}{4}} c^{\frac{1}{4}} l^{\frac{1}{8}}}{W^{\frac{1}{2}}} \cdot x^n$$

the exponent n having the value given in (40).

Replacing x by its value $3 B \frac{\lambda}{\tau} \frac{(W l)^{\frac{1}{2}}}{c}$

and writing for brevity

$$M = \frac{2}{3} A (3 B)^{n - \frac{1}{2}} N,$$

we get finally

$$V = M\left(\frac{fa}{\tau}\right)^{\frac{1}{2}}\left(\frac{\tau}{\lambda}\right)^{\frac{1}{2}-n}\frac{w^{\frac{3}{8}}\,\Delta^{\frac{1}{4}}\,c^{\frac{1}{4}-n}\,l^{\frac{1}{8}+\frac{n}{2}}}{W^{\frac{1}{2}-\frac{n}{2}}}. \qquad (41)$$

206. This formula varies with n, that is to say, with the modulus x to which n is related by (40) or $n = \frac{3}{8} \cdot \frac{1-x}{3-x}$.

It may be used, approximately, by attaching to n a constant value, in conditions of loading such that the modulus remains within certain limits.

207. Among the different forms which the monomial formula for the velocity may take, those deserve special attention which correspond to the value of the modulus $\frac{9}{11}$ and $\frac{6}{10}$.

In the former case $n = \frac{1}{8}$ and in the latter $n = \frac{1}{4}$, and the formula becomes when $x = \frac{9}{11}$, $n = \frac{1}{8}$,

$$V = M\left(\frac{fa}{\tau}\right)^{\frac{1}{2}}\left(\frac{\tau}{\lambda}\right)^{\frac{3}{8}}\frac{w^{\frac{3}{8}}\,\Delta^{\frac{1}{4}}\,c^{\frac{1}{8}}\,l^{\frac{3}{16}}}{W^{\frac{7}{16}}}; \qquad (42)$$

and when $x = \frac{6}{10}$, $n = \frac{1}{4}$,

$$V = M\left(\frac{fa}{\tau}\right)^{\frac{1}{2}}\left(\frac{\tau}{\lambda}\right)^{\frac{1}{4}}\frac{w^{\frac{3}{8}}\,\Delta^{\frac{1}{4}}\,l^{\frac{1}{4}}}{W^{\frac{3}{8}}}, \qquad (42a)$$

which expressions agree with those previously obtained (§ 192).

208. The former of these expressions is applicable to what are called quick, and the latter to slow powders in the scale (§ 200).

209. Since the velocity increases continually as τ decreases, the value τ_1, which gives a maximum, ought to be considered as the superior limit for the use of the Binomial formula.

Consequently, this formula should not be used for powders quicker than the powder of the maximum. This takes

place when the second term of the function $1 - B\beta \frac{(W\,l)}{c}$ is greater than the value $\frac{1}{3}$ which corresponds to the maximum.

It is practically advantageous to limit the use of the formula to cases where the modulus is below a less limit than unity. For higher values, τ becoming nearly equal to τ_1, the theoretical expression becomes too rapidly stationary, and ceases to represent exactly the real variation of the velocity.

If $\frac{9}{11}$ be adopted as the superior limit of the modulus, the binomial formula should cease to be used when the value of the second term $B\beta \frac{(W\,l)^{\frac{1}{4}}}{c}$ is found to be greater than $\frac{1}{3}$ of $\frac{9}{11}$, or greater than $0 \cdot 273$.

210. When the modulus is greater than $\frac{9}{11}$ the monomial formula is applicable.

In fact this formula agrees sensibly with the other, for values of the modulus approaching $\frac{9}{11}$, and increases gradually with the modulus, instead of passing through a maximum. It may, therefore, represent exactly the velocity, in all cases where the powders used act as quick powders, and this has been verified experimentally by M. Sarrau, by comparing the calculated velocities with those actually obtained under very varying conditions of loading.

211. Making use of the characteristics α and β the formula (42) may be written

$$V = M\,\alpha\,\beta^{-\frac{3}{8}} \frac{w^{\frac{3}{8}}\,\Delta^{\frac{1}{4}}\,c^{\frac{1}{8}}\,l^{\frac{3}{16}}}{W^{\frac{7}{16}}}, \qquad (43)$$

and to obtain the value of M, it is sufficient to observe the velocity given by a powder of which the characteristics are known, under given conditions of fire.

Table of the Function f (x).

212. The function $f(x)$ which serves to express the relation of the velocity as a function of the modulus, is represented by $f(x) = \frac{1}{2} x^{\frac{1}{2}} (3 - x)$ when x is less than $\frac{9}{11}$, and by $N x^{\frac{1}{3}}$ when x is greater than $\frac{9}{11}$.

The constant N is determined by equating these two expressions, making $n = \frac{9}{11}$.

The following table gives the value of $f(x)$ for increasing values of x from 0·5 to 1·2.

213.

Modulus x.	$f(x)$.	Log $f(x)$.
0·5	0·8839	− 1·94639
0·6	0·9295	− 1·96825
0·7	0·9622	− 1·98325
0·8	0·9839	− 1·99293
0·9	0·9986	− 1·99939
1·0	1·0118	0·00511
1·1	1·0240	0·01028
1·2	1·0352	0·01501

214. From (37) it is seen that when the duration of combustion of a powder is altered, all other ballistic elements remaining unchanged, the velocity varies directly as $f(x)$, and from (38) and (39) under like circumstances, the pressure on the breech and on the projectile varies as the modulus itself.

Consequently the preceding table affords the means of comparing the corresponding values of the pressure and velocity; and it shows, that the increase of velocity is very small compared with the increase of pressure.

For instance, comparing a powder of modulus 0·6 with another of modulus 1·2, the pressure is doubled whilst the velocity is only increased by about $\frac{1}{8}$th part.

Influence of the Ballistic Elements on the Velocities and Pressures.

215. If c the calibre, l the travel of the projectile, and W the weight of the projectile be given, then, with the same value of f and the same form of grain, by varying the value of the variables w, Δ, and τ, the weight of charge, gravimetric density, and time of combustion, an infinite number of systems may be found in which the velocity will be the same whilst the maximum pressure varies, or the maximum pressure the same whilst the velocity varies.

216. As has been already shown, the maximum pressure is not the same on the base of the projectile as on the breech, and as the latter is always the greatest, and it is the greatest which chiefly concerns the design of a gun, attention will be directed in the following remarks only to the pressure on the breech.

Total Variations of the Velocity and Maximum Pressure.

217. Let it be required to find the variation of velocity and of maximum pressure corresponding to small increments of the variables w, Δ, and τ.

Taking the logarithms of (37) and (39) and differentiating, we get

$$\frac{dV}{V} = \frac{3}{8}\frac{dw}{w} + \frac{1}{4}\frac{d\Delta}{\Delta} + \frac{f_1(x)}{f(x)} \cdot dx \tag{44}$$

and

$$\frac{dP_0}{P_0} = \frac{3}{4}\frac{dw}{w} + \frac{d\Delta}{\Delta} + \frac{dx}{x}. \tag{45}$$

Denoting by τ_1 the duration of combustion, which in the gun

under consideration corresponds to the maximum velocity, we have $x = \frac{\tau_1}{\tau}$ where τ_1 is independent of w and Δ; therefore

$$\frac{dx}{x} = -\frac{d\tau}{\tau} \quad \text{or} \quad dx = -\frac{x}{\tau} d\tau.$$

Making use of which in (44) and (45), and making as before

$$n = x \frac{f_1(x)}{f(x)} \text{ (§ 205)} \tag{46}$$

we get

$$\frac{dV}{V} = \tfrac{3}{8}\frac{dw}{w} + \tfrac{1}{4}\frac{d\Delta}{\Delta} - n\frac{d\tau}{\tau} \tag{47}$$

and

$$\frac{dP_0}{P_0} = \tfrac{3}{4}\frac{dw}{w} + \frac{d\Delta}{\Delta} - \frac{d\tau}{\tau}. \quad (48$$

Now by (§ 207) $n = \frac{1}{8}$ when the modulus is $> \frac{9}{11}$, and it is given by (40) when it is less than $\frac{9}{11}$.

It increases when the modulus decreases below $\frac{9}{11}$, it is equal to $\frac{1}{4}$ when the modulus $= \frac{6}{10}$ corresponding to a very slow powder.

Variation of Velocity corresponding to a Constant Value of Maximum Pressure.

218. By means of the above equations (47) and (48) we may examine how the velocity varies, by the variation of the weight of charge, gravimetric density, and time of combustion, whilst at the same time the maximum pressure remains unchanged.

219. (*a*) *Let the weight of charge be constant, gravimetric density and time of combustion variable.*

Since (48) P_0 is constant $\frac{d\,P_0}{P} = 0$, also $d\,w = 0$; therefore $\frac{d\,\tau}{\tau} = \frac{d\,\Delta}{\Delta}$, substituting which in (47)

$$\frac{d\,V}{V} = \tfrac{1}{4}\,\frac{d\,\Delta}{\Delta} - n\,\frac{d\,\Delta}{\Delta} = (\tfrac{1}{4} - n)\,\frac{d\,\Delta}{\Delta}.$$

If the modulus $> \frac{6}{10}$ $n < \frac{1}{4}$ and $\frac{1}{4} - n$ is positive, therefore the velocity increases with the gravimetric density. From which the following proposition is derived.

When the weight of charge remains the same, and the gravimetric density and the time of combustion increase, so that the maximum pressure remains unaltered, the velocity is increased, and the more so as the modulus of the powder is greater, or the powder quicker.

From which it follows, that by using a very quick powder, and at the same time decreasing its gravimetric density, the velocity may be increased without increasing the pressure.

220. (*b*) *Let gravimetric density* Δ *be constant, and the weight of charge, and time of combustion variable.*

Since $d\,P_0$ and $d\,\Delta$ in (47) and (48) are each $= 0$

$$\frac{d\,V}{V} = \tfrac{3}{4}\,(\tfrac{1}{2} - n)\,\frac{d\,w}{w}.$$

Now from (40) it is seen that $n = \frac{1}{2}$ for $x = 0$, and $n < \frac{1}{2}$ for any other value of x, consequently $\frac{1}{2} - n$ is always positive, from which is deduced the following proposition :—

When the gravimetric density is constant, and the weight of charge, and time of combustion increase so as to keep the maximum pressure constant, the velocity is increased.

221. (*c*) *Let* τ *be constant,* w *and* Δ *variable.*

Then $d\,P_0$ and $d\,\tau = 0$, and

$$\frac{\delta\,V}{V} = \tfrac{1}{4}\,\frac{d\,w}{w}\,. \qquad (49)$$

Therefore, *When, with the same powder, the weight of charge increases, and the gravimetric density decreases, so that the pressure remains unchanged, the velocity is increased.*

222. (*d*) ***When the capacity of the chamber is constant, and w and τ variable.***

Let S = capacity of chamber, then

$$\Delta = \frac{w}{S} \quad \text{and} \quad \frac{d\Delta}{\Delta} = \frac{dw}{w};$$

therefore (47) and (48) become

$$\left.\begin{aligned} \frac{dV}{V} &= \tfrac{5}{8}\frac{dw}{w} - n\frac{d\tau}{\tau} \\ \frac{dP_0}{P_0} &= \tfrac{7}{4}\frac{dw}{w} - \frac{d\tau}{\tau} \end{aligned}\right\} \qquad (50)$$

If now w and τ vary so that P_0 remains constant, we get

$$\frac{d\tau}{\tau} = \tfrac{7}{4}\frac{dw}{w},$$

from which

$$\frac{dV}{V} = \tfrac{7}{4}\left(\tfrac{5}{14} - n\right)\frac{dw}{w}. \qquad (51)$$

Now in ordinary conditions of practice $n < \frac{5}{14}$ so that V increases with w, therefore

In a given gun when the charge and duration of combustion increase so that the pressure remains constant, the velocity is increased.

Consequently, with a size of chamber sufficiently large, the velocity may be increased without altering the pressure by increasing the charge of a powder for which τ is greater.

223. (*e*) From (50) we may determine the variation of velocity corresponding to a small variation of the duration of combustion.

Suppose w remains constant, then

$$\frac{dV}{V} = -n\frac{d\tau}{\tau}. \qquad (52)$$

n increases as the modulus decreases, therefore, the same relative variation of the duration of combustion has an influence on the velocity, the greater as the time of combustion is less.

Corresponding Variation of the Velocity and Maximum Pressure.

224. (*f*) Let the duration of combustion receive a small variation $d\,\tau$, all the other elements remaining constant. Then from (50)

$$\frac{d\,P_0}{P_0} = -\frac{d\,\tau}{\tau}, \tag{53}$$

and eliminating $d\,\tau$ between (52) and (53)

$$\frac{d\,V}{V} = n\frac{d\,P_0}{P_0}. \tag{54}$$

The value of n is given by (40) when the modulus is less than $\frac{9}{11}$, and is equal to $\frac{1}{8}$ when the modulus is greater than $\frac{9}{11}$.

Limiting Values of Modulus.

225. It has already been stated that the modulus should be confined within the limits $\frac{6}{10}$ and $\frac{9}{11}$. The reason for that limitation is this, that when the modulus exceeds $\frac{9}{11}$ the relative increase of the velocity is only $\frac{1}{8}$ the relative increase of the pressure. This inconvenience is diminished when the modulus decreases below $\frac{9}{11}$, because the value of n increases, but then the relative variation of the velocity corresponding to the same relative variation of the duration of combustion increases according to (52), so that the influence of accidental irregularities in the powder, upon the velocity, increases continually. It is therefore necessary to fix an inferior limit to the modulus, in order to insure sufficient regularity in the velocity, and this limit is fixed by M. Sarrau at $\frac{6}{10}$.

226. The superior limit of $\frac{9}{11}$ may no doubt be, and often is exceeded in actual practice. In fact, cases often occur in

which the modulus is greater than unity, but M. Sarrau considers such conditions unfavourable in general. It is, however, obvious that this is a question which depends chiefly on the strength of the gun, and therefore by increasing this, higher ballistic effects will be obtained with quick than with slow powders.

227. In the reception of powder in France a certain margin is allowed which is called "tolerance."

In the manufacture of powder some irregularities are unavoidable, so that different lots of the same powder give different velocities at proof. The limit of these velocities is fixed and is designated by the term "tolerance."

By the foregoing formulæ, the influence which this "tolerance" exercises on the velocity may be estimated.

Suppose for example that the irregularity is due to a variation in the duration of combustion, and let τ be the duration of combustion, which for a given form of grain, gives the normal velocity at proof, that is to say the mean of a great number of fires, and suppose that with a particular lot of powder, this duration receives a variation of $d\tau$, then the corresponding variation of velocity in any gun is given by formula (52).

Let n_0 be the value of n in the éprouvette and V_0 the mean velocity of reception, then

$$\frac{dV_0}{V_0} = -n_0\frac{d\tau}{\tau};$$

consequently,

$$\frac{dV}{V} = \frac{n}{n_0}\cdot\frac{dV_0}{V_0}. \tag{55}$$

Suppose then that dV_0 represents the maximum deviation allowed at reception, the relation (55) gives the difference of velocity which results from it in another gun. If then ϵ_0 denotes the difference of the limits of reception, and ϵ the maximum difference of velocities in any gun, we have

$$\epsilon = \frac{nV}{n_0V_0}\cdot\epsilon_0\,. \tag{56}$$

It is to be remembered that n is expressed in function of the modulus according to (40), and that when the modulus $> \frac{9}{11}$ we must according to (42) take $n = \frac{1}{8}$.

228. Suppose for example, that the French powder $W^{\frac{13}{16}}$, is received for the gun of 24 mm. (9·5 inches) with the conditions following—

$$V_0 = 441 \text{ m.} \qquad \epsilon_0 = 9 \text{ m.} \qquad x_0 = 1{\cdot}194. \qquad n_0 = \tfrac{1}{8}.$$

If the same powder be used in a gun of 10 mm. (= 4 inches) $W = 12$ kilog., $l = 226$ dm., to obtain a velocity of 485 m. We have

$$V = 485 \qquad x = 0{\cdot}642 \qquad n = 0{\cdot}255;$$

therefore by (56) we find

$$\epsilon = 20^{\text{m}} \cdot 2$$

which is the difference from the velocity 485 due to the irregularity which in the 24 mm. gun only gave a deviation of 9 m. from 441 metres.

On the Constants contained in the Equations for Velocity and Pressure.

229. The equation for velocity is

$$V = A\,a\,(w\,l)^{\frac{3}{8}} \left(\frac{\Delta}{W\,c}\right)^{\frac{1}{4}} \left\{1 - B\,\beta\left(\frac{W\,l^{\frac{1}{4}}}{c}\right\}\right..$$

If then the value of the characteristics a and β are known for any particular powder, the constants A and B are easily obtained by firing two rounds, with the same powder, but with a variation in the ballistic elements, for in this way two equations would be obtained containing the two unknown quantities.

230. Now it is evident from the form of the above equation that we may assume arbitrarily values of a and β from any one powder which may be called the "Type powder," and for any other powder may find the relative values of

a and β. This is what M. Sarrau has done, and he has chosen for the Type powder the powder known in France as $W\frac{13}{16}$, that is to say a Wetteren powder of which the thickness is 10 mm. and the sides of the bases 13 and 16 mm. respectively. The density $\delta = 1 \cdot 794$, and the number of grains to the kilogramme or N = 330 to 385, and for this powder he assumes the values $f = 1$, $\tau = 1$. The composition of this powder is

Saltpetre	75·0
Sulphur	12·5
Charcoal	12·5

231. As will be seen hereafter, the values of a and λ for this powder are $a = 2 \cdot 572$ and $\lambda = 0 \cdot 851$, values depending entirely on the *form* of grain. 2.394 7832

And by Table (§ 319)

$$\log a = 0 \cdot 20513$$
$$\log \beta = -1 \cdot 92993$$

232. Making use of these values and of the mean observed velocities obtained with this powder under the service conditions of firing in a 10 cm. (4 inch) and 19 cm. (7·6 inch) gun, in the equation for the velocity (13) the values of A and B were found to be

$$\log A = 3 \cdot 16767$$
$$\log B = -2 \cdot 18373$$

The following are the ballistic elements from which the above values were determined.

Nature of gun.	c	l	W	w	Δ	V
19 cm. gun	1·94	32·9	75	15	0 870	4480
10 cm. gun	1·00	22·6	12	3·1	0·957	4850

Unities, kilogrammes and decimetres.

The value of $\tau = 1$ assumed, is that of the actual time of combustion of one grain of $W\frac{13}{16}$ at the velocity of burning of 10 mm. per second.

Determination of a and λ.

233. It has already been shown (§ 161) that the general form of $\psi(t)$ the ratio of the powder burnt at the end of the time t to the total weight of the grain is

$$\psi(t) = a\frac{t}{\tau}\left(1 - \lambda\frac{t}{\tau} + \mu\frac{t^2}{\tau^2}\right)$$

when

$$a = 1 + x + y, \qquad d = \frac{x + y + xy}{1 + x + y}, \qquad \mu = \frac{xy}{1 + x + y},$$

$$x = \frac{a}{\beta}, \qquad y = \frac{a}{\gamma},$$

and a, β, and γ the three dimensions of the grain, of which a is the least.

Determination of a and λ for Spherical Grains.

234. Here if R = radius of grain, we have the volume of grain at beginning $= \frac{4}{3}\pi R^3$.

Ditto at end of time $t = \frac{4}{3}\pi(R - vt)^3$.

Volume burnt in t

$$= \frac{4}{3}\pi\left\{R^1 - R^3\left(1 - \frac{vt}{R}\right)\right. ;$$

but

$$v = \frac{R}{\tau}, \quad \text{or} \quad \frac{v}{R} = \frac{1}{\tau},$$

therefore volume burnt in t

$$= \frac{4}{3}\pi R^3\left\{1 - \left(1 - \frac{t}{\tau}\right)^3\right.$$

and ratio of volume burnt to original volume or

$$\psi(t) = \frac{1 - \left(1 - \frac{t}{\tau}\right)^3}{1} = 3\frac{t}{\tau}\left(1 - \frac{t}{\tau} + \frac{t^2}{3\tau^2}\right).$$

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Comparing which with the general formula

$$\psi(t) = a\,\frac{t}{\tau}\left(1 - \lambda\frac{t}{\tau} + \mu\frac{t^2}{\tau^2}\right)$$

we get

$$a = 3; \qquad \lambda = 1; \qquad \mu = \tfrac{1}{3}.$$

Cubical Grain.

235. Here as in the first place $\alpha = \beta = \gamma$, and therefore

$$a = 3; \qquad \lambda = 1; \qquad \mu = \tfrac{1}{3}.$$

Small Irregular Grain.

236. These may be considered as spherical grains of which the mean radius is found as follows.

N = number of grains per kilogramme.
δ = absolute density of powder.
R = mean radius of grain.

Then

$$\tfrac{4}{3}\pi R^3 \delta N = 1 \quad \text{or} \quad R = \left(\frac{3}{4\pi\delta N}\right)^{\frac{1}{3}},$$

and consequently as above

$$a = 3; \qquad \lambda = 1; \qquad \mu = \tfrac{1}{3}.$$

237. *Flat Grain, Dimensions α, β, and γ, of which α is the least.*

Here

$$a = 1 + x + y$$

$$\lambda = \frac{x + y + xy}{1 + x + y}$$

$$\mu = \frac{xy}{1 + x + y}.$$

Suppose then

$$x = \frac{\alpha}{\beta} = \frac{1}{1\cdot 3}, \qquad y = \frac{\alpha}{\gamma} = \frac{1}{1\cdot 2};$$

substituting which values we get

$$a = 2\cdot633; \qquad \lambda = \cdot8502; \qquad \mu = \cdot2419.$$

Cylindrical Grain with Round Hole.

238. Let R = external radius of grain.
r = radius of hole.
h = height of grain.

Then, original volume

$$= \pi (R^2 - r^2) h.$$

Volume burnt at t

$$= \pi (R^2 - r^2) h - \pi (R^2 - \rho^2) h \left(1 - \frac{2vt}{R - r}\right)\left(1 - \frac{2vt}{h}\right),$$

and if $R - r$ be less than h

$$\frac{R - r}{2} = vt \quad \text{or} \quad \frac{2v}{R - r} = \frac{1}{\tau}, \quad \text{and} \quad \frac{2v}{h} = \frac{1}{\tau}\frac{(R - r)}{h};$$

and writing

$$x = \frac{R - r}{h}$$

volume burnt at t

$$= \pi (R^2 - r^2) h - \pi (R^2 - r^2) h \left(1 - \frac{t}{\tau}\right)\left(1 - x\frac{t}{\tau}\right),$$

and the ratio of this to the original volume is

$$\psi(t) = 1 - \left(1 - \frac{t}{\tau}\right)\left(1 - x\frac{t}{\tau}\right) = (1 + x)\frac{t}{\tau}\left(1 - \frac{x}{1 + x}\cdot\frac{t}{\tau}\right);$$

therefore

$$a = 1 + x; \qquad \lambda = \frac{x}{1 + x}; \qquad \mu = 0.$$

Prismatic Grain with Hole.

239. This may be treated as a cylindrical grain, using instead of R, the mean radius of the inscribed and circumscribed circles of the hexagon.

On the Relation between the Duration of Burning of a Powder and its Physical Properties.

240. If e and τ be the thickness and time of burning of a grain of powder, and v the corresponding rate of burning

$$\tau = \frac{e}{2v};$$

and since according to Piobert's experiments the velocity is inversely as the absolute density, we have for grains differing only in thickness and density

$$\tau = \kappa \, \delta \, e,$$

κ being a constant.

241. M. Sarrau, adopting this relation, has applied it to the calculation of the "Characteristics" of various powders, but a comparison of the values of τ thus obtained, with those obtained by actual experiments, shows that the above formula does not exactly represent the law according to which the time of burning depends on the thickness and density.

In fact, in the case of two powders $W\frac{32}{38}$ and $W\frac{25}{30}$, of the same composition and nearly the same density, the ratio of the thickness being $1 \cdot 5$, that of the time of combustion is $1 \cdot 25$, and for two other powders SP_3 and SP_2, of the same composition, the ratio of the thickness was $1 \cdot 84$, whilst that of the time of combustion was $1 \cdot 53$.

Again, if the duration of combustion was exactly proportional to the density, the velocities given by different powders would be inversely as the $\frac{1}{4}$th power of the density, but experience shows that the variation of the velocity is considerably greater than would be given by this law.

Consequently, we are led to the opinion that the actual duration of combustion increases more rapidly than the density, and less rapidly than the thickness.

242. It is easy to perceive that this may be due in great measure to the process of manufacture. In some cases, the amount of compression may be such as to ensure a near approach to uniformity of density throughout the cake

whilst in others, the interior of the cake may be less dense, and thus after the outer crust is burnt through, the combustion may go on much more rapidly.

It would be necessary therefore in such cases, to introduce a mean velocity of burning, which would be a function of the thickness of the grain.

243. An empirical formula representing approximately the relation between the time of burning and the thickness would be of much practical use, but to establish it numerous and careful experiments would be required.

244. Meanwhile, M. Sarrau adopts provisionally the following formula

$$\tau = \kappa \left(\frac{e}{1{\cdot}875 - \delta}\right)^{\frac{1}{2}}$$

in which κ varies according to the process of manufacture, and he gives for the French powders in the following tables, which he divides into three categories, the following values.

Powder.	Saltpetre.	Sulphur.	Charcoal.	Mode of Manufacture.	Absolute Density.	Value of κ.
W. Wetteren	75·5	12·0	12·5	Ground and pressed	1·79	0·915
C. and S.P.	75·0	10·0	15·0	Ditto	1·735-1·780	0·665
A.S. $\frac{30}{40}$ ..	75·0	10·0	15·0	Ditto	1·800	0·860

245. With these values of κ M. Sarrau has calculated the values of τ for the following powders.

Name of Powder.	Dimensions of Grains.	Grains to the Kilogramme.	Value of τ.		Difference.
			From Experiment by Firing.	From Formula.	
	mm.				
$W\frac{13}{16}$	10 × 13 × 16	330 to 385	1·000	0·972	+ 0·028
$W\frac{20}{25}$	16 × 20 × 25	104 to 116	1·292	1·294	– 0·002
$W\frac{25}{30}$	20 × 25 × 30	55 to 60	1·546	1·547	– 0·001
$W\frac{34}{38}$	30 × 34 × 48	18	1·942	1·966	– 0·024
C_1	6·6 × 8 × 14·5	1900	0·511	0·470	+ 0·041
C_2	.. ..	..	0·554	0·535	– 0·001
SP_1	9 to 10·3 × 13 × 20	360	0·714	0·722	– 0·008
SP_2	12 to 13 × 17 × 21	110	0·932	0·934	– 0·002
SP_3	23 to 24 × 35 × 35	20	1·423	1·442	– 0·019

246. The differences are inconsiderable except for the $W\frac{13}{16}$ and C_1, and are accounted for by M. Sarrau as due to some difference in the mode of manufacture. The formula therefore gives very approximate results, but should be applied to other powders under reserve, in the absence of further verification.

247. Although the formula $\kappa \delta e$ is acknowledged to be inexact, it has nevertheless given satisfactory results when compared with those actually obtained by firing.

It is however preferable, in determining τ from the physical properties of a powder, to make use of the formula

$$\tau = \kappa \left(\frac{\epsilon}{1 \cdot 875 - \delta}\right)^{\frac{1}{2}},$$

taking the value of κ from the table given above; but as even this formula is uncertain owing to the uncertainty of the law which connects κ with the composition and mode of manufacture, it is always best to deduce the value of τ from ballistic results actually observed.

For this it suffices to measure the velocity obtained under certain ballistic conditions.

For instance, the value of H is obtained from the monomial relation

$$V = \frac{H w^{\frac{3}{8}} \Delta^{\frac{1}{4}} l^{\frac{3}{16}} c^{\frac{1}{8}}}{W^{\frac{7}{16}}},$$

and then τ is obtained from

$$H = M \left(\frac{f a}{\tau}\right)^{\frac{1}{2}} \left(\frac{\tau}{\lambda}\right)^{\frac{3}{8}};$$

or denoting by the suffix $_0$ the data relative to the type powder $W\frac{13}{16}$, the relation of $\frac{\tau}{\tau_0}$ is given by the relations

$$\frac{\tau}{\tau_0} = \left(\frac{a}{a_0}\right)^4 \left(\frac{\lambda_0}{\lambda}\right)^3 \left(\frac{H_0}{H}\right)^8.$$

248. In this latter formula, replacing the factors of the type powder $W\frac{13}{16}$ by their numerical values, we get

$$\tau = N a^4 \lambda^{-3} H^{-8}$$

whence

$$\log N = 27 \cdot 95399.$$

It must however be borne in mind, that the above formula for V ceases to be applicable when the powder acts in the éprouvette as a slow powder; in which case recourse must be had to the binomial formula for the velocity.

The Determination of the "Characteristics" of a Powder.

249. The characteristics of a powder are represented by

$$a = \left(\frac{f\,a}{\tau}\right)^{\frac{1}{2}} \quad \text{and} \quad \beta = \frac{\lambda}{\tau}.$$

It has already been shown that for powders of approximately the same composition the value of f does not vary much, and therefore if any one powder be selected as a type powder, we may for that powder make f equal unity, its actual numerical value being included in the constants A and M.

For the type powder chosen by M. Sarrau, $W\frac{13}{16}$, the thickness of the grain is 10 mm., and taking the velocity of combustion as determined by Piobert to be in free air 10 mm. per second, we get $\tau = 1$.

Now a and λ are determined by the form of grain as shown above (§ 233).

250. Consequently, for the type powder, we have $f = 1$, $\tau = 1$, $a = 2 \cdot 572$, $\lambda = \cdot 850$,

For any other powder, the value of τ may be obtained thus, making $f = 1$:

$$V = \left(\frac{A}{\tau}\right)^{\frac{1}{2}} (w\,l)^{\frac{3}{8}} \left(\frac{\Delta}{W\,c}\right)^{\frac{1}{4}} \left\{1 - B\frac{\lambda}{\tau}\left(\frac{W\,l}{c}\right)^{\frac{1}{2}}\right\}$$

and

$$V = M\left(\frac{a}{\tau}\right)^{\frac{1}{2}}\left(\frac{\lambda}{\tau}\right)^{-\frac{3}{8}} \frac{w^{\frac{3}{8}}\,\Delta^{\frac{1}{4}}\,c^{\frac{1}{8}}\,l^{\frac{3}{16}}}{W^{\frac{7}{16}}}.$$

251. The second of these is easily solved for τ; writing

$$X = \frac{M\,w^{\frac{3}{8}}\,\Delta^{\frac{1}{4}}\,c^{\frac{1}{8}}\,l^{\frac{3}{16}}}{V\,p^{\frac{7}{16}}}$$

we get

$$\tau = \frac{a^4\,X^8}{\lambda^3}.$$

The first equation is not directly soluble, but it may be put under the form

$$V\,\tau^{\frac{3}{2}} - X\,\tau = -\,X\,Y$$

where

$$X = A\,a^{\frac{1}{2}}(w\,l)^{\frac{3}{8}}\left(\frac{\Delta}{W\,c}\right)^{\frac{1}{4}}$$

and

$$Y = \frac{B\,\lambda\,(W\,l)^{\frac{1}{2}}}{c},$$

and from this τ may be obtained by approximation.

252. But we do not know *a priori*, which of the above equations is applicable when we fire a powder whose characteristics are unknown, in a given gun. To obviate this difficulty the following method may be used.

253. The monomial formula is applicable when $\gamma = B\frac{\lambda}{\tau}\left(\frac{W\,l}{c}\right)^{\frac{1}{2}}$ is greater than $\cdot 273$; if $\gamma < \cdot 273$ the binomial formula is to be used.

Moreover the two formulæ will give nearly the same results in the vicinity of conditions which make $\gamma = \cdot 273$.

This being so, apply first the formula

$$\tau = \frac{a^4\,X^8}{\lambda^3}$$

and with the value of τ thus obtained find the value of γ. If this value be $> 0\cdot 273$ the monomial formula is actually applicable, and the value of τ thus obtained is to be admitted.

254. If the value of γ however be $< 0 \cdot 273$ it is the binomial formula which should be used, and the value of τ found is only approximate.

Let τ_0, then, be the approximate value, and τ the real value sought.

Substituting τ_0 in the binomial formula for V, we get an approximate value V_0 for the velocity.

Now the binomial formula is of the form $V = f(\tau)$, and developing the second member by Taylor's theorem,

$$V = f(\tau_0) + (\tau - \tau_0) f^1(\tau_0) + \&c.$$

Limiting the development to the first two terms and observing that $f(\tau_0) = V_0$, we get

$$\tau - \tau_0 = \frac{V - V_0}{f^1(\tau_0)}.$$

Now $f^1(\tau_0)$ is the differential coefficient $\frac{dV}{d\tau}$ when τ is represented by τ_0.

But by formula (52)

$$\frac{dV}{d\tau} = -n\frac{V}{\tau},$$

the value of n being given as a function of the modulus x by the relation

$$n = \tfrac{3}{2} \cdot \frac{1 - x}{3 - x}.$$

Moreover by (35) x and γ are connected by the relation $x = 3\gamma$, γ being $= B\beta \frac{(Wl)^{\frac{1}{2}}}{c}$.

Consequently,

$$f^1(\tau_0) = -\tfrac{1}{2} \cdot \frac{V_0(1 - 3\gamma)}{\tau_0(1 - \gamma)}$$

and

$$\tau - \tau_0 = -2\,\frac{\tau_0(1 - \gamma)}{V_0(1 - 3\gamma)} \cdot (V - V_0),$$

which approximation is generally sufficient.

Application of Formula to the Designing of Guns.

PROBLEM I.

255. Given the calibre and weight of projectile, to determine the conditions to be adopted to realise a given initial velocity and a given maximum pressure.

256. The maximum pressure allowable is fixed by the resistance of the gun, and is the pressure at the breech. It is therefore this pressure which must be introduced into the formula.

On account, however, of the greater simplicity of the formula obtained, M. Sarrau resolves the problem with regard to the maximum pressure on the base of the projectile, and then transforms the results, so as to introduce the maximum pressure on the breech.

257. The calibre and weight of projectile being given, the variables disposable to obtain the internal velocity V_2 and maximum pressure on projectile P, are l, w, Δ, f, a, λ, τ, the first three of which relate to the gun, and the last three to the powder.

If then f is determined by the mode of fabrication of the powder, a and λ by the form of grain, the number of variables is reduced to four, l, w, Δ, and τ.

Now

$$\mathrm{V} = \mathrm{A}\left(\frac{fa}{\tau}\right)^{\frac{1}{2}}(wl)^{\frac{3}{8}}\left(\frac{\Delta}{\mathrm{W}c}\right)^{\frac{1}{4}}\left\{1 - \mathrm{B}\frac{\lambda}{\tau}\frac{(\mathrm{W}l)^{\frac{1}{2}}}{c}\right\}$$

and

$$\mathrm{P} = \mathrm{K}\frac{fa}{\tau}\Delta\frac{(\mathrm{W}w)}{c}\ ;$$

and since V and P are given we have two equations for resolving the problem.

258. If two of the variables w, l, Δ, τ be assumed, the other two may be determined so as that the velocity and pressure may have the required values.

259. The two equations above, are only soluble when l and τ are assumed or known, and w and Δ are the unknown quantities.

It is, however, possible in all cases to put the unknown quantities under an explicit form by taking the modulus of the powder as an auxiliary variable.

For this, it is necessary to consider the relations (37) and (38) which give V and P in function of the modulus x, and the variables l, w, Δ, and add the relation (35) which exists between the modulus and the variables l and τ.

Thus there are three equations:—

$$V = \tfrac{2}{3} A (3 B)^{-\frac{1}{2}} \left(\frac{f a}{\lambda}\right)^{\frac{1}{2}} \frac{w^{\frac{3}{8}} \Delta^{\frac{1}{4}} c^{\frac{1}{4}} l^{\frac{1}{8}} f(x)}{W^{\frac{1}{2}}}$$

$$P = K (3 B)^{-1} \frac{f a}{\lambda} \cdot \frac{\Delta w^{\frac{1}{2}}}{c\, l^{\frac{1}{2}}} \cdot x$$

$$x = 3 B \frac{\lambda}{\tau} \cdot \frac{(W l)^{\frac{1}{2}}}{c}$$

which give the solution of the problem.

260. *Having given the modulus and one of the four variables* l, ω, Δ, and τ, *to determine the other three, so that the initial velocity and maximum pressure have the required values.*

Practically, the gravimetric density Δ is fixed within narrow limits, it is sufficient then to consider this variable as given. In consequence the problem finally resolves itself into this:—

261. *Having given the modulus and the gravimetric density of the charge, to determine the weight of the charge, the length of travel of the projectile, and the time of combustion of the grain, so that the initial velocity and maximum pressure may have given values.*

It is to be remarked that in the above the modulus appears as given with an arbitrary value. It may, therefore, be chosen *a priori*, so that its value shall be within suitable limits as mentioned in (§ 225).

262. The problem being thus fixed as above, its solution is easily obtained from (37), (38), and (35).

The two first equations give w and l. To eliminate l it suffices to multiply together (37) and (38) raised respectively

to the powers 2 and $\frac{1}{2}$. The value of w is thus obtained. From (38) we then get the value of l, and finally τ from (35).

Thus,

$$\frac{w}{W} = H_1\left(\frac{fa}{\lambda}\right)^{-\frac{3}{2}} \frac{V^2 P^{\frac{1}{2}}}{\Delta} \phi(x) \tag{57}$$

$$l = H_2\left(\frac{fa}{\lambda}\right)^{\frac{1}{2}} \frac{\Delta W V^2}{c^2 P^{\frac{3}{2}}} \psi(x) \tag{58}$$

$$\tau = H_3 \frac{\lambda (W l)^{\frac{1}{2}}}{c} \cdot \frac{1}{x} \tag{59}$$

where

$$\left.\begin{aligned} H_1 &= \tfrac{9}{4} A^{-2} (3B)^{\frac{3}{2}} K^{-\frac{1}{2}} \\ H_2 &= \tfrac{9}{4} A^{-2} (3B)^{-\frac{1}{2}} K^{\frac{3}{2}} \\ H_3 &= 3B \end{aligned}\right\} \tag{60}$$

$$\left.\begin{aligned} \phi(x) &= x^{-\frac{1}{2}} f(x)^{-2} \\ \psi(x) &= x^{\frac{3}{2}} f(x)^{-2} \end{aligned}\right\} \tag{61}$$

The values of A, B, and K have been previously given.

$$\begin{aligned} \log A &= 3 \cdot 16767 \\ \log B &= -2 \cdot 18373 \\ \log K &= 3 \cdot 96197 \end{aligned}$$

from which are derived

$$\begin{aligned} \log H_1 &= -10 \cdot 02713 \\ \log H_2 &= 0 \cdot 62937 \\ \log H_3 &= -2 \cdot 66085 \end{aligned}$$

263. Suppose now that instead of P the maximum pressure on the base of the projectile, P_0 the maximum pressure on the breech is given.

In this case, the formulæ are derived from the preceding by replacing P by P_0, and K by $K_0 \left(\frac{w}{W}\right)^{\frac{1}{4}}$ and we get

$$\left(\frac{w}{W}\right)^{\frac{9}{8}} = K_1 \left(\frac{fa}{\lambda}\right)^{-\frac{3}{2}} \frac{V^2 P_0^{\frac{1}{2}}}{\Delta} \phi(x) \tag{62}$$

$$l = K_2 \left(\frac{fa}{\lambda}\right)^{\frac{1}{2}} \frac{\Delta W V^2}{c^2 P_0^{\frac{3}{2}}} \cdot \left(\frac{w}{W}\right)^{\frac{3}{8}} \psi(x) \tag{63}$$

$$\tau = K_3 \frac{\lambda (l W)^{\frac{1}{2}}}{c} \cdot \frac{1}{x} \tag{64}$$

the constants $K_1\,K_2\,K_3$ being derived from $H_1\,H_2$ and H_3 by replacing K by K_0.

The numerical value of K_0 was given above as $\log K_0 = 4 \cdot 25092$, therefore

$$\begin{aligned} \log K_1 &= -\ 11\cdot88266 \\ \log K_2 &= \quad\ \ 1\cdot06279 \\ \log K_3 &= -\ \ \ 2\cdot66088 \end{aligned}$$

the functions $\phi\ (x)$ and $\psi\ (x)$ being always determined by the relations (61).

264. For values of the modulus less than $\frac{9}{11}$, $f\,(x)$ is represented by the expression (36)

$$f\,(x) = \tfrac{1}{2}\,x^{\frac{1}{2}}\,(3 - x);$$

consequently

$$\phi\,(x) = x^{-\frac{1}{2}} f\,(x)^{-2} = \frac{4}{x^{\frac{3}{2}}\,(3 - x)^2} \qquad (65)$$

and

$$\psi\,(x) = x^{\frac{3}{2}} f\,(x)^{-2} = \frac{4\,x^{\frac{1}{2}}}{(3 - x)^2}. \qquad (66)$$

265. For values of the modulus greater than $\frac{9}{11}$, $f\,(x)$ is represented by an expression of the form $N\,x^{\frac{1}{8}}$, and therefore

$$\phi\,(x) = N^{-2}\,x^{-\frac{3}{4}} \quad \text{and} \quad \psi\,(x) = N^{-2}\,x^{\frac{5}{4}} \qquad (67)$$

where N is determined by making $x = \frac{9}{11}$ in the equation

$$\tfrac{1}{2}\,x^{\frac{1}{2}}\,(3 - x) = N^{-2}\,x^{\frac{3}{4}}. \qquad -\frac{3}{4}$$

266.

TABLE OF $f\ (x)$, $\log f\ (x)$, $\log \phi\ (x)$, $\log \psi\ (x)$, AND $\log \dfrac{1}{x}$ FOR VALUES OF x FROM $1\cdot2$ TO $0\cdot5$.

x	$f(x)$	$\log f(x)$	$\log \phi\,(x)$	$\log \psi\,(x)$	$\log \frac{1}{x}$
1·2	1·0352	0·01561	- 1·93039	0·08875	- 1·92082
1·1	1·0240	0·01028	- 1·95874	0·04152	- 1·95861
1·0	1·0118	0·00511	- 1·98978	- 1·98978	0·0000
0·9	0·9986	- 1·99939	0·02410	- 1·93258	0·04576
0·8	0·9839	- 1·99293	0·06259	- 1·86877	0·09691
0·7	0·9622	- 1·98325	0·11095	- 1·80115	0·15490
0·6	0·9295	- 1·96825	0·17442	- 1·73072	0·22185
0·5	0·8839	- 1·94639	0·25773	- 1·65567	0·30103

267. The expressions (57), (58), and (59) give the values of w, l, and τ as functions of V, P, x, and Δ; it remains to determine the laws uniting the unknown with the known quantities. These laws are very simple when the maximum pressure on the breech is amongst the latter; they are somewhat less so when it is the maximum pressure on the breech which is given; but in both cases the general drift of the formula is the same, and we may therefore confine ourselves to the first case in order to study the separate influence of each variable.

For this purpose, in the expressions (57), (58), and (59) substituting the value of l in the equation for τ, and writing

$$\mathrm{H}_4 = \tfrac{3}{2}\,\mathrm{A}^{-1}\,(3\,\mathrm{B})^{\frac{3}{4}}\,\mathrm{K}^{\frac{3}{4}}$$

and

$$\chi(x) = x^{-\frac{1}{4}} f(x)^{-1}$$

we obtain

$$w = \mathrm{H}_1 \left(\frac{f a}{\lambda}\right)^{-\frac{3}{2}} \frac{\mathrm{W}\,\mathrm{V}^2\,\mathrm{P}^{\frac{1}{2}}}{\Delta}\,\phi(x) \tag{68}$$

$$l = \mathrm{H}_2 \left(\frac{f a}{\lambda}\right)^{\frac{1}{2}} \frac{\mathrm{W}\,\mathrm{V}^2\,\Delta}{c^2\,\mathrm{P}^{\frac{3}{2}}}\,\psi(x) \tag{69}$$

$$\tau = \mathrm{H}_4 \left(\frac{f a}{\lambda}\right)^{\frac{1}{4}} \lambda\, \frac{\mathrm{W}\mathrm{V}\Delta^{\frac{1}{2}}}{c^2\,\mathrm{P}^{\frac{3}{4}}}\,\chi(x). \tag{70}$$

268. Hence it appears, that in a gun of a given calibre, for given values of the maximum pressure, modulus, and gravimetric density,—

(*a*) *The weight of charge, and the length of travel of the projectile are proportional to the vis viva of the projectile.*

That in a gun of given calibre and for fixed values of the weight of projectile, modulus, and gravimetric density,—

(*b*) *The weight of charge is proportional to the square root of the maximum pressure.*

The length of travel is inversely as the $\frac{3}{2}$ power of the maximum pressure.

The duration of combustion of a grain is inversely as the $\frac{3}{4}$th power of the pressure.

269. From this it follows, 1stly, that an increase in the strength of the gun, permitting a higher maximum pressure, enables us, with a powder of the same modulus, and the same gravimetric density, to obtain the same velocity, by diminishing the length of the gun, increasing the weight of charge, and using a quicker powder.

270. 2ndly, That in a gun of given calibre, and with fixed values of the weight of projectile, velocity, maximum pressure, and modulus,

The weight of charge is inversely as the gravimetric density.

The length of travel is directly as the gravimetric density.

The duration of combustion of a grain is as the square root of the gravimetric density.

271. Consequently, by enlarging the powder chamber, we can realise, with the same modulus, the same ballistic effect, by decreasing the length of travel, increasing the charge, and using a quicker powder.

272. When all the ballistic elements remain constant except the modulus, the weight of the charge, the length of travel, and the duration of combustion vary directly as the functions $\phi(x)$ $\psi(x)$ and $\chi(x)$ the variation of which is shown in the following table.

273.

x	$\phi(x)$	$\Delta\phi(x)$	$\psi(x)$	$\Delta\psi(x)$	$\chi(x)$	$\Delta\chi(x)$
1·2	·852	·057	1·227	·126	·923	·031
1·1	·909	·068	1·101	·124	·954	·034
1·0	·977	·080	0·977	·121	·988	·040
0·9	1·057	·098	0·856	·117	1·028	·047
0·8	1·155	0·136	0·739	·106	1·075	·061
0·7	1·291	0·203	0·633	·095	1·136	·086
0·6	1·494	0·316	0·538	·085	1·222	·023
0·5	1·810		0·453		1·345	

274. Now, since the weight of the charge is as $\phi(x)$ and the travel as $\psi(x)$, and since $\phi(x)$ increases and $\chi(x)$ decreases as the modulus decreases, it is evident that the same ballistic

effect may be obtained with a slower powder, that is to say a powder of a lower modulus, by a simultaneous *increase* of the weight of charge, and *decrease* of the length of travel.

Influence of the Nature of the Powder and the Form of Grain.

275. The weight of charge and travel of the projectile depend on the nature of the powder and form of the grain, that is to say, on the factor $\frac{fa}{\lambda}$.

The weight of the charge is inversely as the $\frac{3}{2}$ power of $\frac{fa}{\lambda}$, and the length of travel directly as the square root of the same. Consequently, by whatever means $\frac{fa}{\lambda}$ can be increased, the same ballistic result may be obtained with a smaller charge and increased length of travel, employing at the same time a slower powder.

276. The value of $\frac{fa}{\lambda}$ may be increased either by adopting a powder of different composition, such as the picrates, or by the use of forms of grain giving a higher value to a, and a lower value to λ, such as a flat or pierced cylindric grain.

Having given two different guns, to find the relations necessary between the weight of charge, the length of travel, and the gravimetric density, in the two guns, so as to obtain with given moduli, the same velocity and maximum pressure with the same powder.

277. Let P_0 be the maximum pressure on the breech, and V the velocity. Then P_0 and V must be the same in the two guns.

Let c, W, w, l, Δ, x be the elements for one gun,
c', W′, w', l', Δ', x' the corresponding elements for the other;

therefore from (62), (63), and (64) we have for the two guns the following equations:—

$$\left(\frac{w}{W}\right)^{\frac{9}{8}} = K_1 \left(\frac{f\,a}{\lambda}\right)^{-\frac{3}{2}} \frac{V^2\,P_0^{\frac{1}{2}}}{\Delta} \phi(x). \tag{71}$$*

$$\left(\frac{w'}{W'}\right)^{\frac{9}{8}} = K_1 \left(\frac{f\,a}{\lambda}\right)^{-\frac{3}{2}} \frac{V^2\,P^{\frac{1}{2}}}{\Delta'} \phi(x'). \tag{72}$$

$$l = K_2 \left(\frac{f\,a}{\lambda}\right)^{\frac{1}{2}} \frac{\Delta\,W\,V^2}{c^2\,P^{\frac{3}{2}}} \left(\frac{w}{W}\right)^{\frac{3}{8}} \psi(x). \tag{73}$$

$$l' = K_2 \left(\frac{f\,a}{\lambda}\right)^{\frac{1}{2}} \frac{\Delta'\,W'\,v^2}{c'^2\,P_0^{\frac{3}{2}}} \left(\frac{w'}{W'}\right)^{\frac{3}{8}} \psi(x'). \tag{74}$$

$$\tau = K_3 \frac{\lambda\,(W\,l)^{\frac{1}{2}}}{c\,x} \tag{75}$$

$$\tau = K_3 \frac{\lambda\,(W'\,l')^{\frac{1}{2}}}{c'\,x'}. \tag{76}$$

The powder being the same, f, a, λ, and τ do not change.

From (71)

$$\frac{\left(\frac{w'}{W'}\right)^{\frac{9}{8}}}{\left(\frac{w}{W}\right)^{\frac{9}{8}}} = \frac{\Delta\,\phi(x')}{\Delta'\,\phi(x)}. \tag{77}$$

From (72)

$$\frac{l'}{l} = \frac{\Delta'\,W'}{\Delta\,W} \left(\frac{c}{c'}\right)^2 \frac{\psi(x')}{(\psi\,x)} \frac{\left(\frac{w'}{W'}\right)^{\frac{3}{8}}}{\left(\frac{w}{W}\right)^{\frac{3}{8}}}. \tag{78}$$

* For significations of K_1, K_2, K_3, see (§ 263).

From (73)

$$\frac{(W' l')^{\frac{1}{2}}}{c' x'} = \frac{(W l)^{\frac{1}{2}}}{c x} \tag{79}$$

from which we get the values of

$$\frac{\Delta'}{\Delta}, \quad \frac{\left(\frac{l'}{c'}\right)}{\frac{l}{c}}, \quad \text{and} \quad \frac{\frac{w'}{W'}}{\frac{w}{W}}.$$

which relations give the solution of the problem.

278. The equation (79) gives

$$\frac{l'}{l} = \frac{W}{W'}\left(\frac{c' x'}{c x}\right)^2, \tag{80}$$

and giving to $\frac{l'}{l}$ its value from (78) and taking account of the relation (77) we have

$$\frac{\Delta'}{\Delta} = \left(\frac{W}{w'}\right)^3 \left(\frac{c'}{c}\right)^6 \left\{\frac{\psi (x)}{\psi (x')}\right\}^{\frac{3}{2}} \left\{\frac{\phi (x)}{\phi (x')}\right\}^{\frac{1}{2}} \left(\frac{x'}{x}\right)^3; \tag{81}$$

but by (61) $\psi (x) = x^2 \phi (x)$, therefore

$$\left\{\frac{\psi (x)}{\psi (x')}\right\}^{\frac{3}{2}} = \left(\frac{x}{x'}\right)^3 \left\{\frac{\phi (x)}{\phi (x')}\right\}^{\frac{3}{2}} \tag{82}$$

and (81) becomes

$$\frac{\Delta'}{\Delta} = \left(\frac{W}{W'}\right)^3 \left(\frac{c'}{c}\right)^6 \left\{\frac{\phi (x)}{\phi (x')}\right\}^2. \tag{83}$$

Combining (77), (80), and (83), the following formulæ are obtained which give the solution of the problem.

$$\frac{\frac{l'}{c'}}{\frac{l}{c}} = \frac{W c'}{W' c}\left(\frac{x'}{x}\right)^2. \tag{84}$$

$$\frac{\Delta'}{\Delta} = \left(\frac{W}{W'}\right)^3 \left(\frac{c'}{c}\right)^6 \left\{\frac{\phi(x)}{\phi(x')}\right\}^2. \tag{85}$$

$$\frac{\frac{w'}{W'}}{\frac{w}{W}} = \left(\frac{\Delta\,\phi(x')}{\Delta'\,\phi(x)}\right)^{\frac{8}{9}}. \tag{86}$$

279. If the two guns are similar $\frac{W'}{W} = \left(\frac{c'}{c}\right)^3$, and the above relations become

$$\frac{\frac{l'}{c'}}{\frac{l}{c}} = \left(\frac{c\,x'}{c'\,x}\right)^2. \tag{87}$$

$$\frac{\Delta'}{\Delta} = \left(\frac{c}{c'}\right)^3 \left\{\frac{\phi(x)}{\phi(x')}\right\}^2. \tag{88}$$

$$\frac{\frac{w'}{W'}}{\frac{w}{W}} = \left\{\frac{c'\phi(x')}{c\,\phi(x)}\right\}^{\frac{8}{3}}. \tag{89}$$

Application of Formulæ.

280. M. Sarrau proceeds to apply these formulæ to the following problems :—

1. Calculation of initial velocity and maximum pressure in a given gun, under given conditions of firing, with a powder of which the Characteristics are known.
2. Determination of the Characteristics of a powder.
3. Analysis of an existing gun.
4. Determination of the interior dimensions, of the conditions of loading, and of the powder to be adopted, to obtain

a given initial velocity, and maximum pressure, with given calibre and weight of projectile.

5. Determination of the interior dimensions and conditions of loading in order to obtain, with the same powder, a given initial velocity and maximum pressure, in guns of different calibre.

281. These are the principal problems of internal ballistics, and the following is a *résumé* of the notation, employed.

c = calibre of gun.
W = weight of projectile.
l = length of travel.
s = volume of powder chamber.
w = weight of charge.
Δ = gravimetric density ("densité de chargement").
α, β, = Characteristics of the powder.
V = initial velocity.
P = maximum pressure on base of projectile.
P_0 = " at breech.

Unities, decimetre, kilogramme, second.

282. In the following calculations f is taken = 1.

When the problem involves the determination of the powder to be used M. Sarrau adopts the cubical form of grain, in which case

$$\alpha = \left(\frac{f\,a}{\tau}\right)^{\frac{1}{2}} \text{ where } a = 3$$

and

$$\beta = \frac{\lambda}{\tau} \text{ where } \lambda = 1,$$

and the value of τ, the duration of the combustion of a grain in free air, is the unknown which is to be determined.

283. When the form is not cubical, the formula gives a value τ' different from τ, but it is generally useless to calculate τ' directly, when a previous calculation has given τ.

In this case it is enough to determine τ' by the condition that

$$\frac{fa}{\tau'} = \frac{f \times 3}{\tau} \quad \text{or} \quad \tau' = \frac{a}{3}\tau.$$

284. We know that as a consequence of this condition the substitution of any other grain for the cubical grain retains the same maximum pressure and increases the velocity, and therefore the solution which results for the problem is generally more advantageous than that given by the cubical grain, and it may be adopted without adhering too strictly to the exact realisation of the velocity.

285. Having thus found τ, it remains to determine the powder which will realise this duration of combustion.

This would be got with certainty, did we know for each mode of fabrication, the relation existing between the time of combustion of the powder, its thickness, and its absolute density. We could then deduce from this relation, the thickness to be given to the grain for assigned values of the duration of combustion and absolute density. Unfortunately this relation is not at present known. As has been already stated, M. Sarrau adopts provisionally the relation

$$\tau = \kappa\left(\frac{e}{1 \cdot 875 - \delta}\right)^{\frac{1}{2}},$$

where e is the thickness of the grain, δ the density, and κ a constant depending on the mode of fabrication.

M. Sarrau adopts for κ the following values for three descriptions of powder generally used in France, viz.—

1.	Powder W.	.. .. ..	$\kappa = 0 \cdot 915$
2.	„ C. and S.P.	..	$\kappa = 0 \cdot 665$
3.	„ A.S.$\frac{30}{40}$	.. ..	$\kappa = 0 \cdot 860$

Unity of length, the decimetre.

286. Admitting this relation the thickness of the grain is given by the formula—

$$e = (1 \cdot 875 - \delta)\left(\frac{\tau}{\kappa}\right)^2 .$$

287. When the powder used does not differ much from the types actually in service, the following method may be adopted. Having found the value of τ for a cubical grain, find the corresponding value of the characteristic $a = \left(\frac{3}{\tau}\right)^{\frac{1}{2}}$ and compare this with the values of a of the usual powders as given in col. 4 of the table of Characteristics for different powders given in (§ 319). If the value of a obtained is contained between any two values of this table, the powder to be adopted will be intermediate between the two powders to which their value belongs, and this will generally be sufficient to determine the powder to be used.

Problem I.

288. Given

- c the calibre of the gun,
- l the length of travel of the projectile,
- W the weight of projectile,
- w the weight of charge,
- Δ the gravimetric density,

To find

- V the initial velocity,
- P the pressure on the base of projectile,
- P_0 the pressure on the breech.

Let a and β be the Characteristics of the powder.

Calculation of Velocity.

289. Then we have

$$V = A\,a\,(w\,l)^{\frac{3}{8}}\left(\frac{\Delta}{W\,c}\right)^{\frac{1}{4}}(1-\gamma)$$

$$\gamma = B\,\beta\left(\frac{W\,l}{c}\right)^{\frac{1}{2}}$$

$$\log A = 3\cdot16767$$
$$\log B = -\,2\cdot18373.$$

290. The value of γ must first be calculated. If it be less than $0\cdot273$ the above formula is applicable, but if greater we must use the monomial formula

$$V = M\,a\,\beta^{-\frac{3}{8}}\,\frac{w^{\frac{3}{8}}\,\Delta^{\frac{1}{4}}\,c^{\frac{1}{8}}\,l^{\frac{3}{16}}}{W^{\frac{7}{16}}}.$$

when

$$\log M = 3\cdot49425.$$

1*st Example.*

SP_1 powder in 90 mm. gun.

Here

$$c = 0\cdot91,\quad l = 16\cdot7,\quad s = 2\cdot800,\quad W = 8,$$
$$w = 2\cdot4,\quad \Delta = 0\cdot857.$$

	γ.		v.
$\log B =$	$-\,2\cdot18373$	$\log A =$	$3\cdot16767$
„ $\beta =$	$0\cdot07871$	„ $a =$	$0\cdot27926$
„ $W^{\frac{1}{2}} =$	$0\cdot45154$	„ $w^{\frac{3}{8}} =$	$0\cdot14258$
„ $l^{\frac{1}{2}} =$	$0\cdot61136$	„ $\Delta^{\frac{1}{4}} =$	$-\,1\cdot98324$
		„ $l^{\frac{3}{8}} =$	$\cdot45852$
	$-\,1\cdot32534$	„ $(1-\gamma) =$	$-\,1\cdot88511$
Deduct			
$\log c$	$-\,1\cdot95904$		$3\cdot91638$
		Deduct	
$\log\gamma$	$-\,1\cdot36630$	$\log(W\,c)^{\frac{1}{4}}$	$0\cdot21493$
$\therefore\ \gamma =$	$0\cdot23244$		
And $1-\gamma =$	$0\cdot76756$	$\log V =$	$3\cdot70145$

$\therefore\ V = 5029$ dm.
$= 502\cdot9$ metres per second.

291. *2nd Example.*

C_1 powder in 95 mm. gun.

Here

$$c = 0 \cdot 96, \quad l = 19 \cdot 6, \quad s = 2 \cdot 640, \quad W = 10 \cdot 9,$$
$$w = 2 \cdot 1, \quad \Delta = 0 \cdot 795.$$

	γ.		v.
$\log B =$	$-\ 2 \cdot 18373$	$\log M =$	$3 \cdot 49425$
„ $\beta =$	$0 \cdot 26619$	„ $\alpha \beta^{-\frac{3}{8}} =$	$0 \cdot 27206$
„ $W^{\frac{1}{2}} =$	$0 \cdot 52371$	„ $w^{\frac{3}{8}} =$	$0 \cdot 12082$
„ $l^{\frac{1}{2}} =$	$0 \cdot 64613$	„ $\Delta^{\frac{1}{4}} =$	$-\ 1 \cdot 97509$
	———	„ $c^{\frac{1}{8}} =$	$-\ 1 \cdot 99779$
	$3 \cdot 61976$	„ $l^{\frac{3}{16}} =$	$0 \cdot 24229$
Deduct			———
$\log c$	$-\ 1 \cdot 98227$		$4 \cdot 10230$
	———	Deduct	
$\log \gamma$	$3 \cdot 63749$	$\log W^{\frac{7}{16}} =$	45825
$\therefore \gamma =$	$\cdot 43400$		———
$\therefore$ monomial formula must be used.		$\log V$	$3 \cdot 64405$
		$\therefore V = 4406$ dm.	
		$= 440 \cdot 6$ metres per second.	

292. *Calculation of Maximum Pressure.*

P = pressure on base of projectile.
P_0 = pressure on breech.

2nd Example.

The formulæ for the pressure are the following:—

$$P = K \alpha^2 \frac{\Delta (W w)^{\frac{1}{2}}}{c^2} \text{ on base of projectile}$$

$$P_0 = K_0 \alpha^2 \frac{\Delta w^{\frac{3}{4}} W^{\frac{1}{4}}}{c^2} \text{ on breech}$$

when

$$\log K = 3 \cdot 96197$$

and

$$\log \mathrm{K}_0 = 4 \cdot 25092$$
$$,, \quad a^2 = 0 \cdot 74377.$$

Making use of which, the values obtained are :—

$$\mathrm{P} = 2120 \text{ kilo. per dm}^2.$$
$$\mathrm{P}_0 = 2717 \quad ,, \quad ,,$$

293. PROBLEM II.

To determine the Characteristics of a Powder.

As has been already shown these depend upon the mode of manufacture, the form, and the time of combustion of a grain.

Proceeding in the way indicated in (§ 251), we have

$$\tau = a^4 \lambda^{-3} \mathrm{X}^8$$

$$\mathrm{X} = \frac{\mathrm{M}\, w^{\frac{3}{8}}\, \Delta^{\frac{1}{4}}\, c^{\frac{1}{8}}\, l^{\frac{3}{16}}}{\mathrm{V}\, \mathrm{W}^{\frac{7}{16}}}$$

$$\gamma = \mathrm{B}\, \frac{\lambda}{\tau} \frac{(\mathrm{W}\, l)^{\frac{1}{2}}}{c}.$$

294. *1st Example.*

To determine α and β for SP_1 fired from a 155 mm. gun. This powder being of a parallelopipedal form we have

$$\alpha = 2 \cdot 584, \quad \lambda = 0 \cdot 856.$$

Let the initial velocity realised be 459 metres or 4590 dm. per second, under the following conditions :—

$$c = 1 \cdot 56, \quad l = 32 \cdot 2, \quad s = 12 \cdot 80, \quad \mathrm{W} = 40,$$
$$w = 8 \cdot 75, \quad \Delta = 0 \cdot 684.$$

Then from the above we find

$$\begin{aligned} \log X &= -\ 1\cdot75041 & & \\ ,,\ \ \tau &= -\ 1\cdot85503 & \text{or}\ \ \tau &= 0\cdot716 \\ ,,\ \ \gamma &= 0\cdot62400 & \text{or}\ \ \gamma &= 0\cdot42074. \end{aligned}$$

This being greater than $0\cdot273$ the value of $\tau = 0\cdot716$ may be admitted.

2nd Example.

295. To determine α and β for the same powder fired in 90 mm. gun. Here the conditions are:—

$$c = 0\cdot91, \quad l = 16\cdot7, \quad s = 2\cdot800, \quad W = 8,$$
$$w = 2\cdot4, \quad \Delta = 0\cdot857.$$
$$V = 502\cdot2 \text{ metres} = 5022 \text{ dm.}$$

From which we find:—

$$\begin{aligned} \log X &= -\ 1\cdot74823 & & \\ ,,\ \ \tau_0 &= -\ 1\cdot83759 & \text{or}\ \ \tau_0 &= 0\cdot688 \\ ,,\ \ \gamma &= -\ 1\cdot38247 & ,,\ \ \gamma &= 0\cdot24125 \end{aligned}$$

which being less than $0\cdot273$, the method of (§ 254) must be adopted.

First the value of V_0 must be obtained by formula (13), making use of the value of τ_0 and γ just found.

This, in the present case, gives $V_0 = 5058$, then by formula (§ 254.)

$$\tau - \tau_0 = -\ 2\,\frac{\tau_0(1-\gamma)}{V_0(1-3\gamma)}\,(V - V_0)$$

we find

$$\tau - \tau_0 = 0\cdot027$$

add

$$\tau_0 = 0\cdot688$$

gives

$$\tau = 0\cdot715$$

which is the same as found in the first example.

Problem III.

296. *Analysis of an existing Gun.*

Under usual conditions the chamber is not entirely filled with the charge.

The charge may therefore be increased, and combining this increase with a slower powder the velocity may be increased without increasing the pressure.

The amount of this increase is however limited by the value of the modulus, which, for reasons already given (§ 225), should not exceed $\frac{6}{10}$.

297. To appreciate the ballistic effect possible to be realised in an existing gun with a fixed maximum pressure, and with the above limit of modulus, the following method is to be adopted.

Let P_0 be the pressure on the breech which must not be exceeded, and which is only bounded by the strength of the gun. The relation (39) gives

$$P_0 = K_0 (3\,B)^{-1} \frac{f\,a}{\lambda} \left(\frac{w}{W}\right)^{\frac{1}{4}} \frac{\Delta\, w^{\frac{1}{2}}}{c\, l^{\frac{1}{2}}} \,.\, x.$$

Substituting $\frac{w}{s}$ for Δ gives w as a function of P_0 and the modulus x.

Giving x successive values decreasing by $0 \cdot 1$, and beginning from the superior limit of $x = 1 \cdot 2$, we find a series of weights of charge realising the same maximum pressure with powders of increasing slowness.

Then by formula (37)

$$V = \tfrac{2}{3}\, A\, (3\,B)^{-\frac{1}{2}} \left(\frac{f\,a}{\lambda}\right)^{\frac{1}{2}} \frac{w^{\frac{3}{8}}\, \Delta^{\frac{1}{4}}\, c^{\frac{1}{4}}\, l^{\frac{1}{8}}}{W^{\frac{1}{2}}} f(x)$$

we get corresponding velocities. And since by (34)

$$x = 3\,B \frac{\lambda}{\tau} \frac{(W\,l)^{\frac{1}{2}}}{c}$$

τ the corresponding time of combustion is found.

298. The least value to be given to the modulus is either $\frac{6}{10}$ or that value superior to $\frac{6}{10}$ which corresponds to the maximum of gravimetric density, which may be taken as unity.

299. The formulæ to be used are

$$\left.\begin{aligned} w^{\frac{7}{4}} &= A_1\left(\frac{fa}{\lambda}\right)^{-1} s\,c\,l^{\frac{1}{2}}.\,W^{\frac{1}{4}}\,P_0\,\frac{1}{x} \\ V &= A_2\left(\frac{fa}{\lambda}\right)^{\frac{1}{2}} \frac{w^{\frac{3}{8}}\Delta^{\frac{1}{4}}\,c^{\frac{1}{4}}\,l^{\frac{1}{8}}}{W^{\frac{1}{2}}} f(x) \\ \tau &= A_3\,\lambda\,\frac{(W\,l)^{\frac{1}{2}}}{c}\,.\,\frac{1}{x} \end{aligned}\right\} \tag{90}$$

where

$$\begin{aligned} \log A_1 &= -\,6\cdot40993 \\ \log A_2 &= 3\cdot66116 \\ \log A_3 &= -\,2\cdot66085, \end{aligned}$$

the values of $f(x)$ and $\frac{1}{x}$ being given in the table § 323.

300. *1st Example.*

24 cm. Naval gun of 1870.

$c = 2\cdot42$, $l = 38\cdot2$, $s = 35$, $W = 144$,
$P_0 = 2500$ kilog. per cm.^{2}, $a = 3$, $\lambda = 1$.

301. The following table gives the values of w, V, and τ, calculated as above, for values of x decreasing by $0\cdot1$ downwards from $1\cdot2$.

x	w	Δ	V	τ	log a
1·2	27·19	0·777	436·2	1·170	0·20453
1·1	28·57	0·816	444·1	1·276	0·18563
1·0	30·17	0·862	455·0	1·403	0·16495
0·9	32·04	0·915	456·4	1·560	0·14206
0·8	34·27	0·929	479·1	1·574	0·11648

302. The last column contains the logarithms of the characteristic $a = \left(\frac{fa}{\tau}\right)^{\frac{1}{2}}$.

By comparing this value with those of the table, it will be seen which of the actual service powders will approximately realise the required ballistic effect.

Values of x below $0 \cdot 8$ have not been used, as in that case Δ would exceed unity.

It will be seen by comparing the above table with table (§ 319), that with a powder approaching to SP_2 a charge of $27 \cdot 19$ kilog. would give a velocity of $436 \cdot 2$ metres per second, whilst with a powder approaching to SP_3 a charge of $34 \cdot 27$ kilog. would increase the velocity to $479 \cdot 1$ metres per second, without increasing the pressure. In order to obtain a higher velocity it would be necessary to increase the size of the chamber and use a still higher charge of slower powder.

303. Similar tables to the above may be found for a series of guns of given dimensions for each form of grain of powder and for a fixed maximum pressure, and from these tables the conditions may be known under which any powder whose characteristics are known, may be used with a fixed maximum pressure.

304. For instance, let it be required to be known, under what conditions the powder $W\frac{20}{25}$ could be used in the 64 cm. gun, so as not to exceed the maximum pressure of 2500 kilog. per cm.^{2}

By table (§ 319), the log a for $W\frac{20}{25}$ is $0 \cdot 16524$, and by table (§ 301) log a is $0 \cdot 16494$, corresponding to $x = 1$, that is to say that with a charge of $30 \cdot 17$ kilog., density $0 \cdot 862$, of a powder nearly approaching to $W\frac{20}{25}$, a velocity of 455 metres per second would be obtained without increasing the pressure.

Problem IV.

305. *Given the calibre and weight of projectile, the initial velocity, and maximum pressure, to find the interior dimensions of the gun, the conditions of loading and the powder to be used.*

306. The formulæ to be used are (71), (72), (73).

$$\left.\begin{aligned} \left(\frac{w}{W}\right)^{\frac{9}{8}} &= K_1 \left(\frac{fa}{\lambda}\right)^{-\frac{3}{2}} \frac{V^2 P_0^{\frac{1}{2}}}{\Delta} \phi(x) \\ \frac{l}{c} &= K_2 \left(\frac{fa}{\lambda}\right)^{\frac{1}{2}} \frac{\Delta W V^2}{c^3 P_0^{\frac{3}{2}}} \left(\frac{w}{W}\right)^{\frac{3}{8}} \psi(x) \\ \tau &= K_3 \lambda \left(\frac{W l}{c}\right)^{\frac{1}{2}} \cdot \frac{1}{x} \end{aligned}\right\} \qquad (91)$$

where

$$\begin{aligned} \log K_1 &= -\ 11 \cdot 88266 \\ \log K_2 &= \quad 1 \cdot 06279 \\ \log K_3 &= -\ 2 \cdot 66085 \end{aligned}$$

The values of $\phi(x)$, $\psi(x)$, and $\frac{1}{x}$ are given in the table.

307. To make use of these formulæ, the following are assumed as known, c, W, V, P_0, Δ; a and λ depending on the form of grain, and f as usual reduced to unity.

This being so, let different values of the modulus be assumed, say 0·9, 0·8, and 0·7, corresponding to a quick, a medium, and a slow powder respectively.

For each of these values of the modulus corresponding values of $\frac{w}{W}$, $\frac{l}{c}$, and τ will be obtained by means of the above equations, and from the three solutions thus obtained that one will be selected which is most suitable.

1st *Example.*

90 cm. gun.

308. The powder is supposed to be cubical, but we may pass to any other form of grain as shown above (§ 283).

Let $c = 0 \cdot 91$, $W = 8$, $V = 450$ m., $P_0 = 2000$, $\Delta = 0 \cdot 950$.

From which the following values are obtained:—

x	$\frac{w}{W}$	$\frac{l}{c}$	τ	log a
0·9	0·183	20·70	0·666	0·22026
0·8	0·198	18·41	0·728	0·30743
0·7	0·219	17·21	0·805	0 28569

309. The solution when $x = 0 \cdot 9$ gives the lowest charge, and if it be adopted we get

$$w = 1 \cdot 464 \text{ kilog.}, \qquad l = 18 \cdot 8 \text{ dm.}$$

and comparing the value of log a with the table (§ 319), we find that the powder $C._2$ approximates very nearly to the required powder.

2nd Example.

42 cm. gun.

310. Here $c = 4 \cdot 2$, $W = 780$, $V = 500$ m., $P_0 = 2500$, $\Delta = \cdot 950$, and the following values are obtained.

x	$\frac{w}{W}$	$\frac{l}{c}$	τ	log a
0·9	0·244	20·19	3·115	$-$ 1·99177
0·8	0·264	17·95	3·306	$-$ 1·97894
0·7	0·291	15 94	3·560	$-$ 1·96282

Supposing the powder to be a medium powder we adopt the solution corresponding to $x = 0 \cdot 8$, and we get

$$w = 205 \text{ kilog.}, \qquad l = 75 \cdot 4 \text{ dm.}$$

311. The table (§ 319) contains no powder suitable under these conditions. We must therefore inquire what thickness of grain will be necessary for a suitable powder.

312. Let the nature of the powder be that of $AS\frac{30}{40}$, and the absolute density be $1 \cdot 830$. Then by the table (§ 285) we take $\kappa = 0 \cdot 860$ and since $\tau = 3 \cdot 306$, we get

$$e = (1 \cdot 875 - \delta)\left(\frac{\tau}{\kappa}\right)^2 = \cdot 045\left(\frac{3 \cdot 306}{\cdot 860}\right)^2 = 0 \cdot 664$$

L

and the unity being the decimetre, this corresponds to a thickness of 66·4 mm.

Consequently, a powder of similar manufacture to $AS\frac{30}{40}$, with a thickness of grain of about 66 mm., will be the powder required.

Problem V.

Given the Weight of Projectile, the Characteristics of the Powder, the Initial Velocity, and Maximum Pressure, to find the Interior Dimensions and Conditions of Loading in Guns of Different Calibres.

313. Let c and c' be the calibre.

W and W' the weight of projectile.

V and P_0 the velocity and maximum pressure.

In the first place we must assume for the first gun calibre c, the gravimetric density Δ, and the modulus x, and find by means of formulæ (84), (85), (86), the values of $\frac{w}{W}$, $\frac{l}{c}$, and τ which give for these guns the solution of the problem. We then choose for the calibre c' a value of the modulus x', and find the corresponding values of $\frac{l'}{c'}$, $\frac{w'}{W'}$, Δ' by the formulæ (84), (85), (86).

314. The value chosen for the modulus should increase with the calibre. Let $c' > c$, then since the weight of projectile is sensibly proportional to the cube of the calibre, the formulæ (87), (88), and (89) are generally applicable, and if we suppose $x = x'$ these formulæ become

$$\frac{\frac{l'}{c'}}{\frac{l}{c}} = \left(\frac{c}{c'}\right)^2, \quad \frac{\Delta'}{\Delta} = \left(\frac{c}{c'}\right)^3 \text{ and } \frac{\frac{w'}{W'}}{\frac{w}{W}} = \left(\frac{c'}{c}\right)^{\frac{8}{3}}$$

and these give for the calibre c' a value of Δ', notably less than Δ, and a value of $\frac{w'}{W'}$, notably greater than $\frac{w}{W}$ which would lead in general to an excessive size of chamber.

If on the contrary $x' > x$ the value of Δ' increases and $\frac{w'}{W'}$ decreases, but then the value of $\frac{l'}{c}$ increases.

Now in fact, the difficulty of realising a length of travel, considerable comparatively to the calibre, increases with the calibre, consequently a solution by virtue of which $\frac{l'}{c}$ is greater than $\frac{l}{c}$ is not satisfactory, and we may assume that the relation $\frac{x'}{x} = \frac{c'}{c}$ fixes the superior limit of x'.

315. For instance let us take $\frac{c'}{c} = \frac{8}{7}$ or $x = 0 \cdot 7, x' = 0 \cdot 8$; then the formulæ (87), (88), and (89) give

$$\frac{l'}{c'} = \frac{l}{c}, \quad \Delta' = 0 \cdot 837\, \Delta, \quad \frac{w'}{W'} = 1 \cdot 061 \left(\frac{w}{W}\right)$$

which represents very nearly the case of the service guns of 32 cm. and 27 cm.

316. If again we take $\frac{c'}{c} = \frac{9}{7}$ or $x = 0 \cdot 7, x' = 0 \cdot 9$

$$\frac{l}{c'} = \frac{l}{c}, \quad \Delta' = 0 \cdot 702\, \Delta, \quad \frac{w'}{W'} = 1 \cdot 147 \left(\frac{w}{W}\right)$$

which is nearly the case for the service guns of 155 mm. and 120 mm.

317. To apply the preceding remarks, let it be required to determine the interior dimensions and conditions of loading to be adopted in two guns of 27 cm. and 32 cm. calibre in order to obtain with the same powder a velocity of 535 metres per second with a maximum pressure of 2400 kilog. per square centimetre.

318. Here we have

$$c = 2 \cdot 76 \qquad W = 216 \qquad V = 535 \text{ m.}$$
$$c' = 3 \cdot 22 \qquad W_1 = 345 \qquad P_0 = 2400 \text{ kilog.}$$

Since $\frac{c'}{c} = \frac{3 \cdot 22}{2 \cdot 76} = \frac{8}{7}$ nearly we may take $x = 0 \cdot 7$ and $x' = 0 \cdot 8$ and fixing Δ at $\cdot 950$ for the 27 cm. gun we find by the application of the above formula

For the 27 cm. gun.

$$\Delta = 0 \cdot 950, \quad \frac{w}{\mathrm{W}} = 0 \cdot 322, \quad \frac{l}{c} = 20 \cdot 70,$$

$$w = 69 \cdot 6, \qquad l = 57 \cdot 2, \qquad s = 73 \cdot 30.$$

For the 32 cm. gun.

$$\Delta' = 0 \cdot 735, \quad \frac{w'}{\mathrm{W}'} = 0 \cdot 367, \quad \frac{l'}{c'} = 19 \cdot 76,$$

$$w = 126 \cdot 6, \qquad l = 63 \cdot 6, \qquad s' = 172 \cdot 4.$$

Supposing the grain to be cubical $\tau = 2 \cdot 381$.

If the powder be of similar manufacture to AS$\frac{30}{40}$, and the density $1 \cdot 820$, the thickness of the grain as given by formula (§ 286) is about 42 mm.

319.

TABLE I.

CHARACTERISTICS OF FRENCH POWDERS.

Designation of Powder.	1	2	3	4	5	6	7
	a	λ	τ	$\log a$	$\log \beta$	$\log a \beta^{-\frac{3}{8}}$	$\log a^2$
$W\frac{12}{16}$	2·572	·851	1·000	·20513	− 1·92993	·23141	·41027
$W\frac{20}{25}$	2·766	·920	1·292	·16524	− 1·85243	·22057	·33049
$W\frac{25}{30}$	2·814	·937	1·546	·13002	− 1·78246	·21161	·26004
$W\frac{32}{36}$	2·840	·945	1·942	·08254	− 1·68719	·19944	·16508
C_1	2·832	·943	·511	·37188	·26619	·27206	·74377
C_2	2·532	·836	·554	·32997	·17868	·26297	·65994
SP_1	2·584	·856	·714	·27926	·07871	·24974	·55853
SP_2	2·262	·734	·932	·19244	− 1·89610	·23141	·38489
SP_3	2·344	·766	1·423	·10838	− 1·73103	·20924	·21676
$AS\frac{30}{40}$	2·600	·862	1·783	·08196	− 1·68424	·20037	·16393

320.

TABLE II.

DESCRIPTION OF THE ABOVE POWDERS.

	Dimensions of Grains.		No. of Grains per Kilo-gramme.	Density.		Composition.
	Thickness.	Other Dimensions.		Gravimetric.	Absolute.	
	mm.					
$W\frac{12}{16}$	10	..	330 to 385	1·07 to 1·13	1·794	Saltpetre, 75·5
$W\frac{20}{25}$	16	..	104 to 116	1·05 to 1·15	1·787	Sulphur, 12·0
$W\frac{25}{30}$	20	..	55 to 60	1·14	1·809	Charcoal, 12·5
$W\frac{32}{36}$	30	..	18	1·15	1·800	
C_1	6·2 to 6·8	8 to 14·5	$<$ 1900	$>$ 0·91	$>$1·738	
C_2	8	..	625 to 650	$>$ 0·91	1·760	Saltpetre, 75
SP_1	9 to 10·3	13 to 20	$<$ 360	$>$ 0·91	$>$1·785	Sulphur, 10
SP_2	12 to 13	17 to 21	$<$ 110	$>$ 0·91	$>$1·800	Charcoal, 15
SP_3	23 to 24	35	$<$ 20	$>$ 0·91	$>$1·815	
$AS\frac{30}{40}$	30	40	13 to 14	1·15	1·800	Saltpetre, 75·5; Sulphur, 12·0; Charcoal, 12·5

TABLE III.

321. DIMENSIONS OF FRENCH GUNS.

Designation of Gun.	1 Calibre.	2 Length of Travel of Projectile.	3 Weight of Projectile.		4 Capacity of Chamber.
	dm.	dm.	kilog.	kilog.	dm. cube.
10 cm. for the Marine (1870)	1·00	22·6	10	12	3·165
14 " "	1·41	27·0	21	28	4·285
16 " "	1·66	30·2	"	45	9·695
19 " "	1·94	32·9	62·5	75	17·25
24 " "	2·42	38·2	120·	144	35·00
27 " "	2·76	41·0	180·	216	52·50
32 " "	3·22	51·6	286·5	345	86·25
34 " "	3·40	48·3	"	425	132·00
80 mm. Land Gun, Mountain	·805	9·6	5·6	"	0·615
80 " " Field ..	·805	17·1	5·6	"	2·073
90 " " " ..	·91	16·7	8·0	"	2·800
95 " " " ..	·96	19·6	10·9	"	2·640
120 " " " ..	1·21	24·8	18·0	"	6·045
155 " " " ..	1·56	32·2	40·0	"	12·80
190 " " " ..	1·94	30·9	75·0	"	21·10
240 " " " ..	2·42	40·5	120·0	"	38·05

Since 1 kilog. of powder at gravimetric density ("densité de chargement") equal one occupies 1 dm. cube, the figures in column 4 represent the weight of charge when the chamber is filled at $\Delta = 1$.

322.

TABLE IV.

MODULI OF FRENCH POWDERS IN THEIR RESPECTIVE GUNS.

Designation of Guns.	Weight of Projectile.	$W\frac{13}{16}$	$W\frac{20}{25}$	$W\frac{25}{30}$	$W\frac{32}{38}$	$AS\frac{30}{40}$	C_1	C_2	SP_1	SP_2	SP_3
10 cm. Marine	10	0·586									
	12	0·642									
14 ,, ...	21	0·658	0·551								
	28	0·760	0·636								
16 ,, ...	45	0·865	0·724	0·616							
19 ,, ...	62·5	0·911	0·760	0·649	0·521	0·517					
	75·0	0·998	0·835	0·711	0·571	0·567					
24 ,, ...	120	1·091	0·912	0·766	0·623	0·620					
	144	1·195	1·000	0·851	0·683	0·678					
27 ,, ...	180	..	1·015	0·864	0·694	0·689					
	216	..	1·112	0·946	0·760	0·755					
32 ,, ...	286·5	..	..	1·048	0·842	0·836					
	345	..	..	1·150	0·924	0·917					
34 ,, ...	425	..	..	..	0·939	0·933					
80 mm. Land Service ..	5·6	..	..	..	..	..	0·770	0·630			
80 mm. Field	5·6	..	..	..	..	..	1·028	0·840	0·667		
90 ,, ...	8·0	..	..	..	..	..	1·074	0·878	0·697		
95 ,, ...	10·9	..	..	..	..	..	1·302	1·064	0·846	0·555	
120 ,, ...	18·0	..	..	..	..	..	..	1·207	0·959	0·630	
155 ,, ...	40·0	..	..	..	..	..	..	..	1·263	0·830	0·567
190 ,, ...	75·0	..	..	..	..	..	..	..	..	0·895	0·612
240 ,, ...	120·0	..	..	..	..	..	..	..	..	1·038	0·710

323. TABLE V.—FUNCTIONS OF THE MODULUS.

x	$\log f(x)$	$\log \phi(x)$	$\log \psi(x)$	$\log \frac{1}{x}$
1·2	·01501	−1·93039	·08875	−1·92082
1·1	·01028	−1·95874	·04152	−1·95861
1·0	·00511	−1·98978	−1·98978	0·00000
0·9	−1·99939	·02410	−1·93258	·04576
0·8	−1·99293	·06259	−1·86877	·09691
0·7	−1·98325	·11093	−1·80115	·15490
0·6	−1·96825	·17442	−1·73072	·22185
0·5	−1·94639	·25773	−1·65567	·30103

REDUCTION TO ENGLISH WEIGHTS AND MEASURES.

Value of Constants.

324. In M. Sarrau's investigations the unities are the kilogramme, decimetre, and second. In order to make use of his formula with the English unities of feet, lbs., and seconds it is therefore necessary to change the value of the constants A, B, K, M, &c.

The following table has therefore been prepared which gives at one view the values of these constants in both notations.

TABLE VI.

Formula where used.	Designation of Constant.	FRENCH.	ENGLISH.
(43) (§ 211)	log M	3·49425	2·84567
(15), (17) (§ 174), (§ 177)	log K	3·96197	0·61174
	log K^0	4·25092	0·90069
(13) (§ 172)	log A	3·16767	2·56634
	log. B	− 2·18373	−2·30964
(60) (§ 262)	log H_1	−10·02713	−7·09377
	log H_2	0·62937	−4·74373
	log H_3	− 2·66085	−2·78676
(62), (64) (§ 263)	log K_1	−11·88266	−8·94944
	log K_2	1·06279	−3·17715
	log K_3	− 2·66085	−2·78676
(90) (§ 299)	log A_1	− 6·40993	−3·88607
	log A_2	3·66116	2·99686
	log A_3	− 2·66085	−2·78676

	P and P_0.	w and W.	c and l.	V.
French unities ..	Kilogs. per square dm. ..	Kilogs.	Decimetre	Dm. per sec.
English unities ..	Tons per sq. in.	Pounds	Inches	Feet "

Characteristics of English Powder.

325. For ordinary English powder, it being of the same composition as the C and SP French powder, the value of f may be considered the same, and a and λ being only dependent on the form of grain, the values of τ, and therefore of a and β, may be determined for each powder in the manner described above.

326. As regards prismatic powder, it is probable that some modification will be required. The time of burning depends upon the least thickness of the grain, which in a prismatic powder is the difference between the radius across the flats and the radius of the central hole. As these grains fit close together in the cartridges, the first ignition is almost confined to the surface of the central holes, but as soon as the projectile moves, the grains separate, and then the whole surface becomes ignited. It is probable that this takes place before any considerable proportion of the charge is burnt, and if so the duration of τ will only be very slightly affected, in other words, the actual value of τ will be slightly greater than given by the previous methods of calculation.

327. The value of f for prismatic brown powder and cocoa powder will probably differ from that for the black powders. The value of f is given (§ 106) by the relation

$$f = \frac{p_0 v_0 T_0}{273}.$$

If then the values of τ_0 and v_0 as given in (§ 87) and (§ 79) be admitted we should have for pebble powder

$$f = \frac{1 \cdot 033 \times 278 \cdot 3 \times 2230}{273} = 2349;$$

and for cocoa powder

$$f = \frac{1 \cdot 033 \times 198 \times 2390}{273} = 1791.$$

Consequently, the value of f, as compared with unity adopted by M. Sarrau, will be for cocoa powder

$$1 \times \frac{1791}{2349} = \cdot 7635.$$

328. If this be so, it would appear, that all other conditions being the same, the velocity with cocoa powder will be 87·32 per cent. of the velocity, with a like charge of black powder of the same size and form of grain.

In order, therefore, to obtain the same velocity, it will be necessary to increase the charge, and as the velocity is proportional to the $\frac{3}{8}$th power of the charge, the charge of cocoa powder would be to that of black powder, as 1·437 to 1, or an increase of 43·7 per cent.

329. These remarks must be taken with great reserve, as the actual facts with regard to the temperature of combustion of cocoa powder are very imperfectly known.

330. The characteristics α and β have been carefully determined for the French powders, so that, by means of M. Sarrau's formula, the ballistic results may be predicted for any gun of which the dimensions, the conditions of firing, and the powder are known.

331. In one of M. Sarrau's works is given a table showing the results of actual firing compared with those of calculations made by the Binomial formula for slow powders, from eleven different guns, varying from 12·2 to 3·7 inches calibre, with eleven different powders, varying from 13 grains to 800 grains to the lb., or from about $\frac{1}{4}$ inch cube to $1\frac{1}{2}$ inch cubes, and with gravimetric densities varying from 0·627 to 1·040.

Out of 40 rounds where the initial velocities varied from 860 to 1960 feet per second, 20 rounds averaged by calculation, 11 feet per second below, 14 rounds 12 feet per second above, and 6 rounds exactly agreed with the observed velocities.

332. Again, calculating by the Monomial formula for quick powders, he gives a table of 81 rounds fired with ten different

powders, from fourteen guns of different calibres and lengths, and with velocities of from 1000 to 1960 feet per second, in which the calculated velocities were, in 16 rounds, exactly the same as, in 42 rounds averaged 12 feet per second below, and in 23 rounds 11 feet per second above the observed velocities.

333. Equally satisfactory were the results of the formula for pressure. Out of 19 rounds, fired with different powders, from guns ranging from 3·2 to 9·2 inches calibre, and when the actual observed pressures varied from 8 to 17 tons per square inch, 2 rounds gave an average of 0·19 tons per square inch in excess, 10 rounds gave an average of 0·47 tons per square inch below, and 7 rounds gave exactly the observed pressures.

334. Moreover, M. Sarrau's formulæ enable us to solve the following problems:—

(*a*) Given, the calibre, weight of projectile, initial velocity, and maximum pressure on breech,

To find, the length of travel of the projectile, the weight and gravimetric density of the charge with any given powder.

(*b*) Given the initial velocity, maximum pressure, and "Characteristics" of the powder,

To find for guns of any calibre, the internal dimensions, the weight and gravimetric density of the charge.

(*c*) To determine the "Characteristics" of any powder.

335. These are the chief problems of internal ballistics and by their aid guns may be designed to give any required ballistic results or *vice versa*, the ballistic results may be predicted for any given gun with any given powder.

336. Unfortunately, not only are the formulæ themselves generally unknown in this country, but the "Characteristics" of our powders have not been determined.

337. From such limited data as have been at my disposal, I have calculated the "Characteristics" for the powders given in the following tables, which, however, must be taken with great reserve until they have been more accurately determined.

Powder.	Guns from which the Characteristics were determined.	a	λ	τ	$\log \alpha$	$\log \beta$	$\log \alpha \beta^{-\frac{3}{8}}$	$\log \alpha^2$
R.L.G.$_2$	13 pr. B.L.	3	1	·5278	·37732	·27753	·27325	·75468
	9 pr. No. 21	3	1	·5675	·36160	·24608	·26942	·72320
	12 pr. No. 9	3	1	·4395	·41709	·35706	·27321	·83418
	Average ..	..	..	·5116	·38409	·29107	·26817	·76818
R.L.G.^{4}	Hotchkiss 3 pr.	3	1	·6629	·32783	·17855	·27799	·65157
		3	1	·6573	·32968	·18224	·26130	·65976
		3	1	·6244	·34083	·20454	·26288	·68168
	Average ..	..	..	·6482	·33270	·18828	·26210	·66541
P.	12-inch Gun	2·776	·9238	·8549	·25594	·03364	·24333	·51188
	4-inch Gun	”	”	·9865	·22466	−1·97048	·22817	·44932
	6-inch Gun	”	”	1·3351	·15895	−1·84010	·21893	·31790
	Average			1·0589	·21316	−1·94738	·23014	·42637
W.A. 1 in.	12-inch 35-ton Gun	3	1	1·0201	·23424	−1·99136	·23748	·46849
W.A. 2 in.	12-inch 35-ton Gun	3	1	1·5330	·14581	−1·81470	·22559	·29162
Brown Prismatic ..	Average in Guns 6″ to 16″·25	1·5	·333	1·88	−1·95090	−1·24868	−1·0000	−1·90180

338. It is especially with regard to brown prismatic powder that the above figures must be taken with reserve, as the data in my possession are very scanty, and there is an additional source of uncertainty regarding the value of f, as the composition of these powders differs from that of the black powders.

Similitude of Guns.

339. Guns are termed *similar* when their lineal dimensions are in the same proportion, and they are said to be similarly loaded when the weights of the charges and projectiles are as the cubes of the calibre, and when the grain of powder has the same form, is of the same composition, and has its least lineal diameter proportional to the calibre.

340. Under these circumstances, the initial velocities and maximum pressure will be the same in all such guns.

341. The truth of this proposition follows from an examination of the formulæ (13) and (15) and it would be rigorously exact, except for one cause. The loss of heat from the absorption by the walls of the gun is proportional to the square of the calibre, and not to the cube, and as it may be considered as a reduction in the value of the charge, it will clearly be relatively greater in small charges than in large ones. The total amount is, however, not very great, and therefore the general principle of similitude may be considered as true, with the reservation that it is not desirable to apply it to guns differing very largely in calibre.

342. It is worthy of remark, that in the construction of guns, and especially of Wire guns, as is shown by the formula given in my 'Treatise on the Application of Wire to the Construction of Ordnance,' the same principle of similitude exists, so that the dimensions and laying-on tensions will be similar in a gun of 12-inch calibre, to those of a gun of 9 inch or 6 inch.

343. In speaking of the laying-on tensions being similar, it

must be understood that the tensions of laying on are the same, at similar radii. For instance, if we have two guns of calibre c and c_1 and if $c_1 = m\,c$, and if the tension at ρ be t in the first gun, then the same tension will be applicable at a radius $m\,\rho$ in the second, and the strains in the two guns under fire and at rest will be the same when the guns are similarly constructed and similarly loaded.

CHAPTER IV.

INTERNAL BALLISTICS IN RELATION TO GUN CONSTRUCTION.

344. In the preceding chapter, attention has been chiefly directed to the action of powder in giving velocity to the projectile, and to the various phenomena presenting themselves during its combustion in a gun. It is now necessary to examine more particularly into its action upon the gun itself.

345. To deal safely with the very high pressure of gunpowder, guns of great strength are required, and various systems of construction have been proposed and practised, but of late years, most gunmakers have sought to reduce the strain upon the gun by the use of powder of such a nature and character, as to give rise to comparatively small pressures, making up for the loss of ballistic effect by increasing the weight of the charge and the length of the gun. They have sought rather to weaken the powder than to strengthen the gun.

346. Colonel Brackenbury, then Superintendent of the Royal Gunpowder Factory, in a lecture read by him at the Royal United Service Institution in 1884, put the matter very well in these words: "We constantly hear that a gun has been produced which will do this or that, yet it is not the gun which does it, but the gunpowder. The gun is only a tube to concentrate the action of the powder and guide the projectile. There is not a single gun actually adopted for service in any country which is not, by its weakness, a hindrance to the full action of the 'Spirit of Artillery.'

When gunmakers say, as they frequently do, that their guns will produce a certain effect, 'provided that a suitable powder be found for it,' they mean 'provided that the strength of the powder be restrained, cribbed, cabined, and confined, to suit the weakness of the gun.' We sometimes see in human life a great and strong spirit tear to pieces a feeble frame which contains it, and we do not say 'What a pity that the spirit is so strong,' but rather, 'How sad that the body is so weak.'

"In the case of artillery we are always subduing and taming the spirit instead of strengthening the body. This may be necessary under existing circumstances, but if so, the circumstances are unfortunate and stand in the way of getting the most value out of the 'Spirit of Artillery.'"

A great deal has been said of recent years about the great improvement in powder, and it is held that this consists chiefly in its slow burning, and that still further improvements may be looked for in this direction. Indeed General Maitland, a few years ago, was so enamoured with this view of the subject that he said,* "We find in the struggle for existence, the guns growing longer and longer to get the best effects from the slow powder, while the powder tends to grow slower and slower to meet the wants of the guns, in accordance with the eternal principle of evolution;" and so impressed was he with this view, that he said further, "A low maximum pressure long sustained is the great desideratum of the artillerist, and no one will attain any measure of ballistic success who fails to recognise this fundamental maxim."

Again, Captain Noble, of Elswick, in a lecture at the Institution of Civil Engineers in April 1884, said, "When I add that with a given weight of gun a higher effect can be obtained, if the maximum pressure be kept within moderate limits, I trust I have said enough to vindicate the correctness

* Lecture at the R. U. S. Inst. on the Heavy Guns of 1884, 20th June, 1884, by Colonel Maitland, R.A.

of the course which the gunmakers of the world have, so far as I know, without exception followed."

347. Now if by "moderate limits" Captain Noble means a maximum pressure of about 17 tons per square inch, I would observe that the "moderation" must have reference to the strength of the gun, and I have no hesitation in saying, that, by the use of steel wire, a gun may be made with the same *margin of safety*, under a pressure of 30 tons, as a forged steel gun of the same weight under a pressure of 17 tons per square inch, and that the same ballistic effect can be obtained from the wire gun with a much less charge of powder.

Resistance to Bursting Strain.

348. M. Sarrau's formula enables us to determine the maximum pressure P_0 in a gun of given calibre, with a given charge and a required initial velocity.

By the Binomial formula, applicable to slow, i.e. large-grained powders, it appears that as regards the first term, the velocity increases directly as the square root of τ, whilst in the second term, which is subtractive, it varies inversely as the $\frac{3}{2}$ power of τ, so that the effect of increasing τ is to decrease both terms, but the second, which is subtractive, more rapidly than the first, whilst by the Monomial formula the velocity increases inversely as the one-eighth power of τ. Consequently with the same weight of charge the velocity must always be less for a greater value of τ, that is to say for a large-grained powder, and the velocity must be made up by an increased weight of charge, or by a greater length of gun.

349. As regards the maximum pressure, it increases as a^2, i.e. as τ, consequently the increase of pressure is relatively greater than that of velocity.

But the maximum pressure is only limited by the safe

resistance of the gun, and therefore it is clear that the highest ballistic effect must be obtainable from the strongest gun, whilst at the same time the gun (if a wire gun) need not be strained *relatively* more than the weaker forged steel gun.

350. M. Sarrau has shown that there is a difference, which may be very considerable, between the maximum pressure against the breech and that against the base of the projectile, and according to his formula

$$P_0 = \frac{K_0 \, a^2 \, \Delta \, W^{\frac{1}{4}} \, w^{\frac{3}{4}}}{c^2}$$

and

$$P = K \, a^2 \frac{\Delta \, (W \, w)^{\frac{1}{2}}}{c^2}$$

where P_0 and P are the pressures in tons as per square inch on the breech and base of projectile respectively;

- Δ, the gravimetric density;
- c, the calibre in inches;
- w and W, the weight of charge and projectile respectively;
- K_0, and K, constants;
- a, a factor depending upon the nature and form of grain of the powder.

Consequently the maximum strain on the powder chamber, determined by the formula

$$P_0 = \frac{K_0 \, a^2 \, \Delta \, W^{\frac{1}{4}} \, w^{\frac{3}{4}}}{c^2}$$

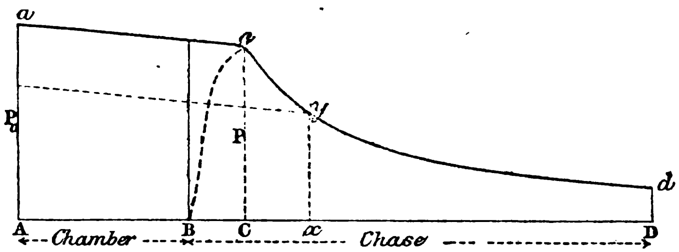
(where $K_0 = 7 \cdot 956$), is the strain to be provided for at the breech end of the chamber.

351. The strain P is not the strain against the base of the projectile in its original position, but the strain when the projectile has moved a certain distance at which the maximum is attained.

This distance is not accurately determinable, but it may be approximately found as will hereafter be shown. It is

small in quick-burning powders, but may be considerable in other cases.

352. The curve showing the maximum pressure is therefore of the following form:—



A B is the powder chamber; B, the position of the base of projectile before it moves; C, the position at the time of maximum pressure on its base; P_0 is the maximum pressure at A; P, the maximum pressure at C. The exact form of the curve between P_0 and P is not known, but for practical purposes it may be taken as a straight line, and the strength of the gun between A and C must be calculated according to the pressure denoted by the ordinates at each point.

When the projectile has reached x, the pressure on its base will have fallen, and may be denoted by the ordinate $x\,y$, and it is probable that the pressures behind this part will approximate to the dotted lines drawn through y parallel to $a\,c$.

The strength of the gun between C and D must be calculated at each point such as x with reference to the ordinate $x\,y$ at that point, and if the form of the curve $c\,y\,d$ be known, the solution of the problem as regards the strength of the gun to resist bursting strain is easy.

353. There is considerable difference of opinion among artillerists respecting the form of the curve representing the pressure on the base of the projectile, and the subject is of such importance that I will devote a little space to its consideration.

354. If a charge of powder, such as R.L.G., or even up to

P, be fired in a gun with due attention to simultaneous ignition, and the products then expand doing work upon the projectile, the pressure will rise very rapidly, and will attain its maximum before the projectile has moved any considerable distance. With such powders it is probable that the whole of the charge is burnt at the time of maximum pressure, and that the work done on the projectile previous to that, is but a small proportion of the total work done.

Subsequent to the time of maximum pressure, the curve will be that of a gaseous fluid, expanding and doing work, subject, however, to modification by the abstraction of heat by the cooling influence of the walls of the gun. Leaving out this modification for the present, the equation to the pressure curve will be given by Noble and Abel's formula:—

$$p = p_0 \left\{ \frac{v_0 (1 - a)}{v - a v_0} \right\}^{\frac{C_p + \beta\lambda}{C_v + \beta\lambda}},$$

where p is the pressure corresponding to volume v;

p_0, the maximum pressure in a close vessel of volume v_0 fired at gravimetric density $= 1$;

C_p, the specific heat at constant pressure;

C_v, the specific heat at constant volume;

a, the ratio of the volume of non-gaseous products to the volume of the powder or v;

β, the ratio between the weights of the non-gaseous and gaseous products of combustion;

λ, the specific heat of the non-gaseous products.

The values of these constants given by Noble and Abel are

$p_0 = 43$ tons* per square inch $= 6554$ atmospheres;

$a = \cdot 57$;

$\beta = 1 \cdot 2957$;

$C_p = \cdot 2324$;

$C_v = \cdot 1762$;

$\lambda = \cdot 45$;

* This is probably greater when the weight of the charge is greater in proportion to the surface of the vessel in which the charge is burnt. It may also probably be in some degree dependent on the composition of the powder.

and the above equation becomes

$$p = 43 \left\{ \frac{\cdot 43}{\frac{v}{v_0} - \cdot 57} \right\}^{1 \cdot 0748}$$

A curve constructed from this formula, I call Noble and Abel's curve, and if a point be taken on it corresponding to the maximum pressure, the ordinates of this curve beyond this point, will represent the pressure on the base of the projectile at the moment when the projectile passes these ordinates.

As regards the pressure curve previous to the time of the maximum, its equation is unknown, and as has already been pointed out when treating of ignition, its determination would be of no great practical use.

355. General Mayewski studied the question from experimental data obtained at Krupp's works in 1867. The times corresponding to the successive passage of the projectile through certain points in the chase were measured. From these a formula was obtained expressing the space as a function of the time. From this by twice differentiating the acceleration and moving force were determined.

356. General Mayewski assumed a formula of the form

$$x = \mathrm{A}\,t + \mathrm{B}\,t^2 + \mathrm{C}\,t^3 + \mathrm{D}\,t^4 \text{ \&c.},$$

and determined the coefficients so as to agree with the mean results of experiment.

Then by differentiation, he got

$$v = \frac{d\,x}{d\,t} = \mathrm{A} + 2\,\mathrm{B}\,t + 3\,\mathrm{C}\,t^2 + 4\,\mathrm{D}\,t^3$$

and by a second differentiation

$$\phi = \text{accel. force} = \frac{d^2\,x}{d\,t^2} = 2\,\mathrm{B} + 6\,\mathrm{C}\,t + 12\,\mathrm{D}\,t^2.$$

The value of t corresponding to the maximum pressure was given by the relation

$$\frac{d^3\,x}{d\,t^3} = o \therefore 6\,\mathrm{C} + 24\,\mathrm{D}\,t = o\,.$$

Thus he found $t = \cdot 0018$ and $x = 4\frac{1}{4}$ inches, or the position of maximum pressure was after the projectile had moved $4\frac{1}{4}$ inches.

These experiments, however, were only made with a 4-pounder gun, and with velocities of about 780 feet per second.

357. Captain Noble, of Elswick, made use of a somewhat different method.

He assumed a function of the form of

$$x = a\,t^{\alpha + \beta t + \gamma t^2},$$

and from the observed values of x and t he determined, by the method of least squares, the probable value of α, β, and γ, taking for unities $\frac{1}{1000}$th of a second and $\frac{1}{1000}$th of a foot.

The formula arrived at was

$$\text{Pebble powder} \begin{cases} x = 3 \cdot 31076\, t^{1 \cdot 378 + 0 \cdot 766 t - \cdot 06932 t^2} & \text{English unities.} \\ x = 1 \cdot 0091\, t^{1 \cdot 378 + 0 \cdot 766 t - \cdot 06933 t^2} & \text{French unities.} \end{cases}$$

$$\text{R. L. G.} \begin{cases} x = \cdot 57837\, t^{3 \cdot 42802 - \cdot 02336 t + \cdot 000245 t^2} & \text{English unities.} \\ x = \cdot 1763\, t^{3 \cdot 42802 - \cdot 02336 t + \cdot 000245 t^2} & \text{French unities.} \end{cases}$$

358. Then by differentiation, the velocity and accelerating force are determined. If this latter be $\phi = \dfrac{d^2x}{dt_2}$ this is the acceleration for $\frac{1}{1000}$th of a second, therefore the acceleration is $1000\,\phi$, and if W be the weight of projectile and P the total pressure over the base, $P = \dfrac{W}{g}\, 1000\,\phi$.

From this formula curves have been constructed showing the velocity and pressure during the early part of the motion of a projectile of 300 lbs. fired from a 10-inch gun, with charges of 70 lbs. P and 60 lbs. R.L.G. respectively, and it is said that these curves represent very approximately the actual results obtained.

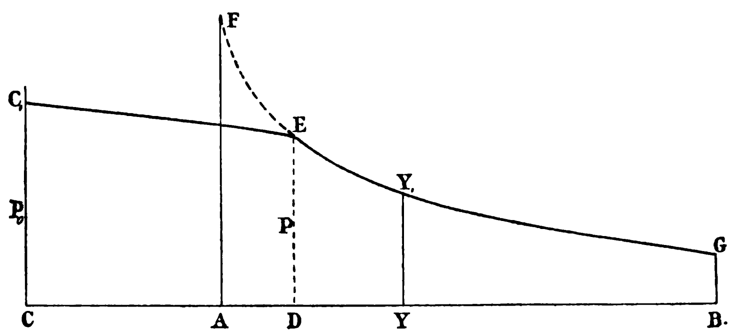
They show that whilst the maximum pressure is acquired with R.L.G. in $0 \cdot 001$ of a second, and when the projectile has moved about $\cdot 05$ of a foot, in the case of the P. powder, the time was about $0 \cdot 0044$, and the distance moved $0 \cdot 45$ of a foot.

359. With the prismatic powders now used in large guns, the projectile moves *relatively* much further before the maximum pressure is attained, and it is this increase of space chiefly that is the cause of the low maximum pressure with such powders.

The form of the curve behind the point of maximum pressure is however of minor importance. What is required, is to know at what point the maximum pressure is attained, since at that point must be the greatest strength of the chase.

For practical purposes it is sufficient to assume that Noble and Abel's curve represents the superior limit of the pressure. If then the maximum pressure on the base of the projectile be calculated by Sarrau's formula, the point on the curve corresponding to the pressure will give the position of maximum pressure, and the curve from that point forward to the muzzle will give the limiting values of the maximum pressure in the chase.

360. The following is the method of applying the preceding observations to practice.



Let C A represent the equivalent length of the chamber of a gun, that is to say the length of a cylinder of the same diameter as the bore of the gun and the same capacity as the actual chamber, and let A B be the length of the chase, or the travel of the shot. Then the maximum pressure will

be in the case of the chamber being filled at gravimetric density = 1.

Then, if the projectile be immovable and the charge fired the pressure will be, according to Noble and Abel, about 43 tons per square inch. Make A F the ordinate at A equal to 43, and then dividing A B into expansions of which A C is the unit, set off along it ordinates such as D E, Y Y_1, B C, representing the corresponding pressures as given by Noble and Abel's formula. The curve F E Y_1 C is what I call Noble and Abel's curve.

By Sarrau's formula calculate P_0 and P, and set off C C_1 = P_0, and take the point E on the curve corresponding to P and join C_1 E. Then C E Y_1 G is the curve of maximum pressure, and its ordinates give the values according to which the strength of the gun must be calculated, so far as regards bursting strain.

361. I do not assert that the actual pressures in the gun are those shown by the curve, but that the curve gives the superior limit, and is consequently a safe guide. The actual pressures in the chase will always be less than those shown by the curve, as there is always a loss of pressure due to the cooling influence of the walls of the gun.

362. There is, I think, considerable misapprehension on this point, to which it is necessary to allude.

Owing, probably, to accidents which have happened to long guns firing large charges of prismatic powder, it has been assumed that the pressures in the forward part of the chase are much higher with slow than with quick-burning powder, and this is said to be due to the continued burning of the powder all, or the greater part, of the time the projectile is in the gun.

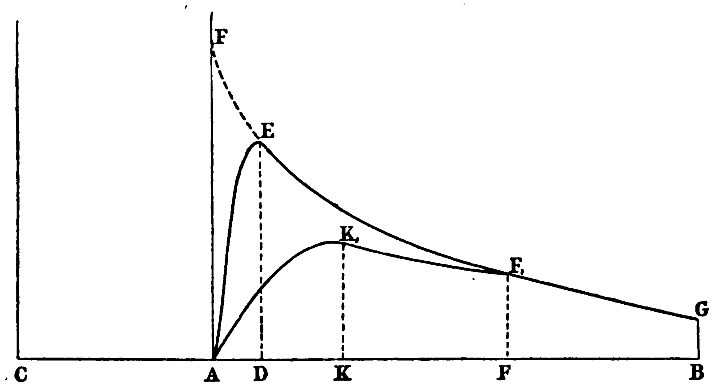
There is no doubt that at times this does take place, and that in some cases a considerable portion of the charge is blown out of the gun unburnt, but this is bad ballistic practice, and I have already shown when treating of the ignition and combustion of powder, that in every case of slow continuous burning, the pressure at any point before the whole

charge is consumed, must be *less* than the pressure at the corresponding point in Noble and Abel's curve.

No doubt the pressures in the forward part of the chase are much greater now than they were in the days of quick powders and shorter guns, but this is entirely due to the enormously increased charges, whereby even a very long gun becomes virtually a short gun, that is to say, a gun of few expansions, and it has nothing to do with the rate of burning of the powder, which only affects the maximum pressure in the vicinity of the chamber.

363. This matter is so important, that, at the risk of repetition it may be well once more to explain it.

Let A C represent the charge of powder fired at gravimetric density 1, and A B the length of the chase, and F E F_1G Noble and Abel's curve, as before described. With a quick



powder, the whole of the powder would be burnt when the projectile arrived at D, and the maximum pressure would be represented by D E, and the pressures corresponding to the motion of the projectile from D to B would be represented by the ordinates of the curve E G.

With a very slow powder, the whole of the charge might not be burnt till the projectile had arrived at F when the pressure would be F F_1 exactly the same as the pressure from

the quick powder at the same point. Consequently the pressure on the chase between F and B would be the same in both cases. It does not follow that $F F_1$ would be the maximum pressure with the slow powder. That might be at a point K where the increasing evolution of gas was exactly balanced by the increasing space behind the projectile, but at this point, and at every point between K and F, the pressures must necessarily be less than with the quick powder for the simple reason that there is a less quantity of powder gas in the same space.

364. It follows, therefore, and this is the important point as regards gun construction, that with equal charges the pressures from slow powders must generally be less, and can never be greater than from quick powders, and that consequently Noble and Abel's curve is a safe guide to the gun constructor as far as the bursting strain is concerned.

365. Since the ordinates of the pressure curves as derived from Noble and Abel's curve, represent the pressure per square inch on the base of the projectile, the area of such curves multiplied by the area of the base of the projectile will give the energy of the shot, and if there were no loss by cooling and nothing expended in the friction and expulsion of the gases, &c., the muzzle velocity would be obtainable from the formula

$$V = \sqrt{\frac{2 g E}{W}} .$$

366. The actual pressure curve as regards the projectile may be obtained from Sarrau's formula for initial velocity; for if v be the initial velocity;

ϕ the accelerating force,
W the weight of the projectile in lbs.,
ω the area of its base,
l the travel of the projectile,

$$\frac{V\,d\,V}{d\,l} \cdot = \phi;$$

and if P_0 be the pressure per square inch

$$P_0 = \frac{W}{\omega\, g} \cdot \phi ;$$

or in tons per square inch.

$$P_0 = \frac{W}{2240\, \omega\, g} \cdot \phi \, .$$

Now by Sarrau's formula

$$V = \frac{A\, a\, (w\, l)^{\frac{3}{8}}\, \Delta^{\frac{1}{4}}}{(W\, c)^{\frac{1}{4}}} \left\{ 1 - \frac{B\, \beta\, (W\, l)^{\frac{1}{4}}}{c} \right\}$$

or

$$V = a\, l^{\frac{3}{8}} (1 - b\, l^{\frac{1}{4}})$$
$$= a\, l^{\frac{3}{8}} - a\, b\, l^{\frac{7}{8}},$$

from which we get the acceleration

$$\phi = \frac{V\, d\, V}{d\, l} = \frac{3\, a^2}{8\, l^{\frac{1}{4}}} - 1 \cdot 25\, a^2\, b\, l^{\frac{1}{4}} + \tfrac{7}{8}\, a^2\, b^2\, l^{\frac{3}{4}},$$

and therefore the pressure in tons per square inch

$$P_0 = \frac{W}{2240\, \omega\, g} \left\{ \frac{\frac{3}{8}\, a^2}{l^{\frac{1}{4}}} - 1 \cdot 25\, a^2\, b\, l^{\frac{1}{4}} + \tfrac{7}{8}\, a^2\, b^2\, l^{\frac{3}{4}} \right\}.$$

Now in Sarrau's formula adapted to English unities (§ 324) $A = 2 \cdot 56634$ and $B = -\ 2 \cdot 30964$; but since g is in feet, l must be taken in feet, or 12 l must be substituted for l in the formula, therefore

$$a = \frac{12^{\frac{3}{8}}\, A\, a\, w^{\frac{3}{8}}\, \Delta^{\frac{1}{4}}}{(W c)^{\frac{1}{4}}} \quad \text{and} \quad b = \frac{12^{\frac{1}{4}}\, B\, \beta\, W^{\frac{1}{4}}}{c}$$

are the constants to be used in the equation for ϕ when the length is taken in feet.

367. As a first application of the formula I take the case of the 6-inch gun built by Messrs. Easton and Anderson from my design, and fired at Woolwich 25th April, 1888, with 100 lbs. projectile and 34 lbs. powder P. gravimetric density $= 1$.

Here

$$\log \alpha = \cdot 21316 \qquad \log \beta = -1\cdot 93843$$
$$c = \text{calibre} = 6 \text{ inches}$$
$$W = 100$$

from which we find

$$a = 1159 \quad \text{and} \quad b = .1023$$
$$\therefore\ V = 1159\, l^{\frac{3}{8}} - 118\, l^{\frac{7}{8}}$$

and making $l = 10\cdot 66$ the total travel of projectile we find $V = 1875$ feet per second, which was very nearly the observed velocity.

368. For the pressure per square inch at any intermediate part x we have, making $l = x$

$$p = \frac{24\cdot 785}{x^{\frac{1}{4}}} - 8\cdot 443\, x^{\frac{1}{4}} + \cdot 604\, x^{\frac{3}{4}}.$$

From which the curve in the following diagram (Fig. 1) has been obtained. ×

The pressures calculated from Sarrau's formulas (17) and (15) are

$$P_0 = 26\cdot 20 \text{ tons per sq. inch.}$$
$$P \ = 17\cdot 70 \quad ,, \quad ,,$$

the latter corresponding to a travel of Projectile of $1\frac{1}{4}$ inches.

The actual pressure given by the crusher gauge was about 25 tons per square inch.

The area of the curve is $83\cdot 3$, which multiplied by 28 the area of the projectile gives the energy 2332 foot-tons, therefore

$$V = \sqrt{\frac{2332 \times 64\cdot 4 \times 2240}{100}} = 1834 \text{ feet per second,}$$

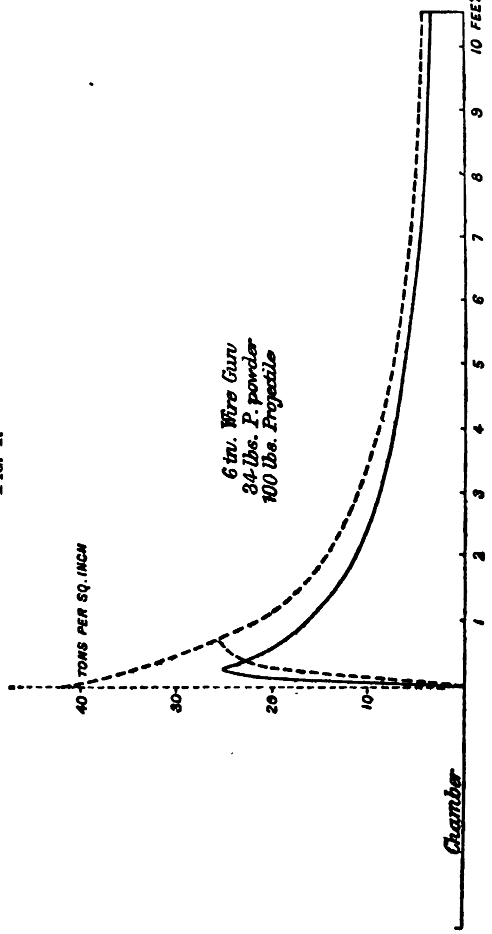
which agrees very nearly with the calculated and the observed velocities.

369. The upper dotted line in the diagram shows the pressure curve according to Noble and Abel's formula, and it will be seen that it is throughout higher than the curve from Sarrau's formula. Taking the area of the upper curve as representing the total energy, that of the lower one the

× This statement is not true.

energy expended on the projectile in giving velocity, it will be found that the latter is about 76½ per cent. of the former, showing that about 23½ per cent. is expended in expelling the

Fig. 1.



gases, friction, &c., which, as will be seen in the last chapter of this book, is probably very near the truth.

370. As a second example I will take a 10-inch gun with a projectile of 500 lbs. and charge of 300 lbs. prismatic brown powder.

In this case

$$\begin{aligned} c &= 12 \text{ inches} \\ l &= 22 \cdot 5 \text{ feet} \\ w &= 300 \text{ lbs.} \\ W &= 500 \text{ lbs.} \\ \log \alpha &= -1 \cdot 98821 \\ ,, \ \beta &= -1 \cdot 26761 \end{aligned}$$

and we find

$$V = 752 \, l^{\frac{3}{8}} - 19 \cdot 17 \, l^{\frac{7}{8}},$$

and when $l = 22 \cdot 5$

$$V = 2125 \text{ feet per second.}$$

The actual velocity observed was 2100 feet per second. The equation for the pressure is found to be

$$p = \frac{18 \cdot 73}{l^{\frac{1}{4}}} - 1 \cdot 6 \, l^{\frac{1}{4}} + \cdot 0289 \, l^{\frac{3}{4}},$$

from which the following diagram (Fig. 2) is obtained.

The maximum pressures calculated by Sarrau's formula are

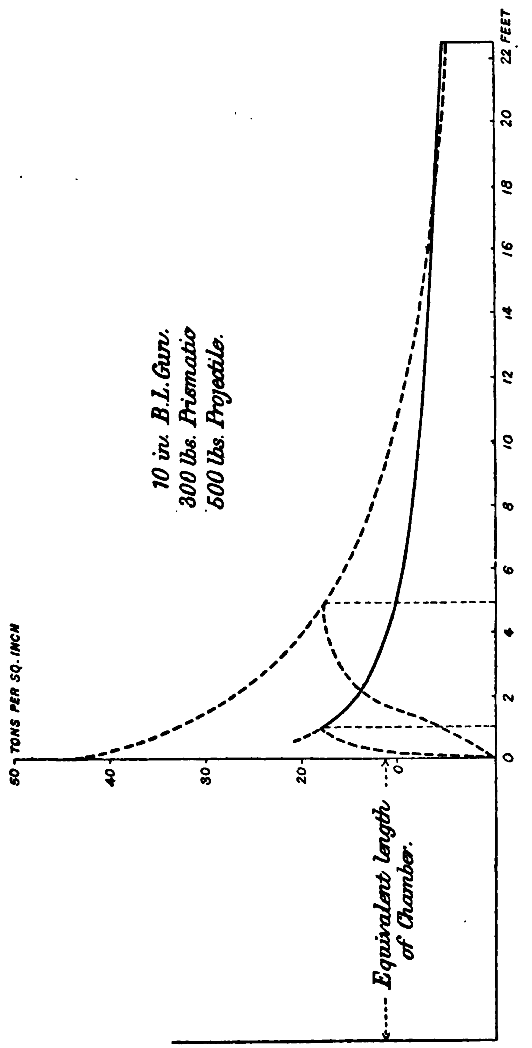
$$\begin{aligned} P_0 &= 17 \cdot 80 \\ P &= 10 \cdot 50. \end{aligned}$$

The pressure by the crusher gauge was 18 tons. The energy calculated from the curve is 15,040 foot-tons, which corresponds to a velocity of 2083 feet per second, which was very nearly the observed velocity.

371. The upper dotted line shows the curve from Noble and Abel's formula for pressure. It will be observed that towards the muzzle it falls slightly below the other curve. This is no evidence against the truth of Noble and Abel's curve. It arises from the value assumed for the index γ in (§ 192) being a mean value approximating from the ordinary conditions of fire; and when the weight of charge was about $\frac{1}{3}$rd of the weight of projectile. The formula is therefore only approximate under the conditions of the second example when the charge was $\frac{3}{5}$ths of the weight of projectile. This leads to a change of the form of curve, although it does not appear to cause any important error in its total area. Calculating the energy from the curve it is found to be 18,533 foot-tons, so that in this case the percentage of energy spent on the projectile is $81 \cdot 15$ per cent., and on the expulsion of gases, friction, and cooling, about $18 \cdot 85$ per cent.

The useful effect is therefore greater in the large gun, which is in accordance with the results of practice, and is due chiefly to the loss by cooling being relatively less.

Fig. 2.



372. From what precedes it is evident that the pressures of Noble and Abel's curve are quite sufficient to give the

observed muzzle velocity to the projectile, as well as to overcome the other internal resistances and the cooling action of the walls of the gun, and consequently a gun whose strength at each portion of the chase corresponds to these pressures, and to the maximum pressure in the chamber as determined by Sarrau's formula, will always be a safe gun as regards the bursting strain.

Longitudinal Strain.

373. There is, however, another strain to be provided for, the Longitudinal Strain.

So far as I know, the only longitudinal strain which has been considered important by artillerists and gun-makers is the strain between the breech and the trunnions.

The maximum value of this strain is at the obturator and if the gun be supposed to be fixed at the trunnions so as to have no recoil, the amount of this strain is $P_0\,\omega$ where P_0 is the maximum pressure and ω the sectional area of the chamber, and in the case supposed of no recoil, the same strain extends to the trunnions.

If, however, the gun is free to recoil, the strain will be gradually diminished by the force required to give acceleration to the mass behind the point at which the strain is calculated, and in this case the strain will be one gradually decreasing from the obturator to the trunnions.

374. The usual way of dealing with this strain is to assume that it is uniformly distributed over the cross-sectional area of the gun. This assumption is entirely wrong. The strain at the obturator is borne unequally according to some law which is not accurately known, but there is good reason to believe that it is analogous to the law which governs the bursting strain, and that it varies inversely as the square of the distance from the axis of the gun. This being so, the inner surface is strained very much more than the average strain on the whole cross-section.

For many years this was disregarded by gun-makers, and the evil was aggravated by throwing this strain directly on the inner tube of the gun, and from that to the breech coil or jacket, through which again it was carried to the trunnions. The condition was still further aggravated by the fact that this portion of the material of the gun had also to sustain the bursting strain. There were consequently two conjugate strains, each of great intensity at the inner surface each to be borne by the same material.

375. So long ago as 1860 I drew attention to this, and advocated the entire separation of these strains, but no regard was paid to the matter, and it is only within the last four or five years that my views have been partially adopted in breech-loading guns by making the breech screw take into the jacket instead of as before into the tube. This is, however, a very partial improvement, inasmuch as owing to the thinness of the inner tube at the breech end, the jacket has to resist a very heavy bursting strain.

376. My opinion has always been that the whole of the bursting strain should be exclusively borne by the tube and its reinforcement, and the whole of the longitudinal strain exclusively by the jacket; and in my very first paper in 1860 I showed how this might be done in the case of wire guns, and yet till within the last few years it has been persistently asserted that this was the greatest difficulty as regards the construction of wire guns. The fact that I had shown how the longitudinal strain was to be provided for, *that I had actually done it in a gun of which the inner tube was of cast iron and only half an inch thick, was quietly ignored*, and it is still constantly asserted that the great difficulty in wire gun construction is to provide for the longitudinal strain.

It has been said by some that my system involves extra weight, that the jacket forms a considerable part of the weight of the gun, and that therefore its material ought to be utilised in increasing the resistance to bursting strain, just as if the material would resist the action of two conjugate strains of equal amount to its tensile strength!

When the strains are kept separate, the precise amount of each being known, the provision by distinct members is accurately determinable, but when they are mixed up so as to act conjointly on one and the same mass of material, as at the breech end of a gun, the problem is exceedingly difficult, and perhaps practically insoluble.

The principle of the separation of the two strains is therefore, in my opinion, one of primary importance.

Longitudinal Strain before the Trunnions.

377. The longitudinal strain between the trunnions and the muzzle does not appear to have been considered of much importance, and yet of late years many cases have happened in which a portion of the chase in front of the trunnion has been fractured.

378. For instance, the 12-inch "Collingwood" gun, which on 10th May, 1886, blew away about 8 feet of the front or muzzle end of the chase.

A Committee of Investigation was appointed, consisting of the Ordnance Committee, with whom were associated the Superintendent of the Royal Gun Factory and members of the Elswick and Whitworth Ordnance Factories. After a long investigation they concluded that the accident was due to—

1st. Want of uniformity in the metal.

2nd. Absence of annealing after forging and hardening, and resulting internal strains.

3rd. Intensification of such strains by firing at proof, and further self-development during the interval of eighteen months between firing at proof and the accident.

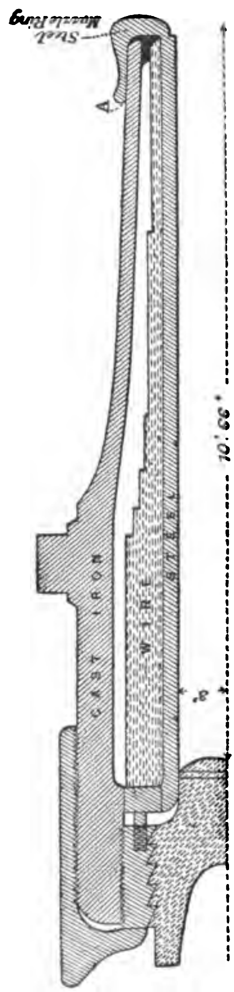
4th. The want of chase-hooping.

They recommended that in these and all other guns of 6-inch calibre and upwards the chambers should be reduced, the service charges reduced, so that the maximum pressure in the chamber should not exceed 15 tons per square inch, and that the chase should be hooped all along to the muzzle.

379. From these recommendations it would appear that

the investigating Committee had nothing in their minds beyond the bursting strain, nor does their report allude in any way to any longitudinal strain as causing or contributing to the cause of the accident.

380. In designing the 6-inch wire gun which was made for the Government by Messrs. Easton & Anderson, and which blew off the muzzle-ring on 25th April, 1888, when fired at Woolwich with 34 lbs. P. powder and 100 lbs. projectile, I had certainly under-estimated the longitudinal strain in front of the trunnions.



6" Wire Gun.—*not to scale.*

A rough sketch of this gun is given in the annexed diagram. The inner tube, which was of Whitworth compressed steel, was 1·6 inch thick at the breech and up to the trunnions, and then gradually tapered to 1·1 inch at the muzzle.

The wire coil was $2\frac{1}{8}$ inches thick over the parallel part of the tube and tapered down to $\frac{1}{2}$ inch at the muzzle end, where it was covered by a thin gun-metal hoop about 4 inches long and $\frac{3}{8}$ inch thick, very lightly shrunk on. The wire coil was supported at the breech end by a steel flange screwed on to the tube in the manner described in my patent of 1885. The outer diameter of this flange exactly fitted a parallel seating bored out in the jacket at that end, but excepting there and at the gun-metal hoop at the muzzle end there was a vacant space between it and the jacket of about $\frac{5}{8}$ inch. From this it will

be seen that the tube and coil were entirely free to move longitudinally in the jacket.

The jacket was of cast iron, and at the muzzle end a steel ring was screwed on, which had a deep flange projecting inwards, against which the steel tube and coil abutted. It was this flange alone which prevented the tube moving forward, and consequently any longitudinal forward force acting on the tube was borne by this flange, and thus transmitted to the jacket at the point A.

At the breech end of the tube, and between it and the breech ring screwed into the jacket, were six set pins, kept up against the flange of the tube by Belville springs, with a forward strain of three or four tons, so that whilst the tube was always kept up to the flange at the muzzle it was free to expand backwards against the springs.

The breech block, it will be seen, is entirely independent of the tube and coil, so that the bursting strain is entirely provided for by the tube and coil, and the longitudinal strain by the jacket.

381. In designing this gun I did not lose sight of the fact that a considerable longitudinal strain would be thrown on the jacket between the muzzle and the trunnions. In the first place there was the pressure of the powder gases on the difference of area between the chamber and the chase, about 1½ square inch. Then there was the friction of the projectile on the grooves, or rather its resultant in the direction of the axis of the gun, and lastly, there was the inertia of the tube and wire coil which had to be set in motion backwards as the gun recoiled. The whole of these strains I had calculated and amply provided for, and yet when the gnn was fired the jacket was torn asunder at the point A, and the steel muzzle ring projected violently to the butts.

382. The sectional area of the jacket at A was 55·8 square inches, and the test pieces cut from it gave a tensile strength at rupture of 16 tons per square inch. If, however, only one-half of this be taken, the rupturing force must have been 446 tons. As will be shown hereafter, the utmost strain

that could arise from the sources above mentioned would not exceed 284 tons, leaving a force of at least 162 tons to be accounted for.

Where did this force come from? On examining the gun and projectile it was found that there had been no jamming, in fact the projectile had been passed through the gun by hand previous to firing, and when fired it went straight to the butt, with a velocity of about 1870 feet per second, the full velocity calculated for the charge of 34 lbs. P. powder. The tube was uninjured, but it had moved bodily forward nearly an inch in the jacket. It was therefore evident that it was the forward motion of the tube which ruptured the jacket, and that that forward motion relative to the jacket had brought into play a force of at least 446 tons, of which I could only account for 284.

383. After a little consideration I came to the conclusion, which subsequent examination of the subject only confirms, that a very important forward longitudinal strain, hitherto altogether unappreciated, was caused by the friction of the products of combustion against the inner surface of the tube. So strong was my conviction of this that a few days after the accident I wrote to the War Office proposing that the gun should be sent back to the makers to be repaired, and that I should have an interview with the Ordnance Committee to explain to them my views and reasons, before they reported on the accident. I also suggested that experiments, the nature of which I was prepared to explain, should be made to set the question at rest, which was the more desirable, inasmuch as it appeared to me to have an important bearing on the future of the Collingwood and other guns.

No notice was taken of my letter. I was not allowed to see the Ordnance Committee, but after about two months, having heard privately that they had reported on the accident, I wrote again to the War Office, asking what was going to be done with the gun, and that I might have a copy of, or be informed of the reasons given by the Ordnance Committee for the accident in their report.

The reply to this was that no further experiment would be made with the gun, and that the report was a confidential document, and I could not be allowed to know its contents!

I have, however, learnt that the Ordnance Committee entirely rejected my view regarding the friction of the products of combustion, and I have no reason to suppose that my suggestion of experiments being made is likely to be entertained.

384. Under these circumstances, and with a deep conviction of the importance of the question, I now submit the following investigations, not as a solution of the problem, but in the hope that it may be the means of directing the attention of others to the subject, who may be much more competent to deal with it than myself, and who may have the means of undertaking such experiments (which are not of a difficult nature), so as to ascertain practically what is the real effect and amount of the friction of the products of combustion on the surface of the chase of a gun.

385. In the first place I will make a few brief remarks, which will perhaps be admitted to contain *primâ facie* evidence that the amount of this friction may be very considerable.

386. The erosion of guns is the work of the products of combustion. These products are not altogether gaseous, a large proportion is generally admitted to be liquid in a very diffused state, but however this may be, there is no doubt that the mixed products do actually cut away particles of solid steel as though they themselves were solid. This cannot be done without exercising a forward force upon the exposed surfaces, because these products are themselves moving forward.

387. Let the gun be considered at the moment of maximum pressure. At this moment the projectile will have moved forward about 10 inches. The surface exposed would be 636 square inches, and the pressure 25 tons per square inch. There is therefore a mass of mixed gases and liquid pressing on a surface of steel, with a pressure of 20,900 tons. At the breech end the mixed mass is at rest, but against the

projectile it is moving with the velocity of the projectile. The mean velocity of the mass is about 318 feet per second, so that there is a mass of gases and liquid pressing with a force of 20,900 tons against the surface of the bore, and at the same time moving forward with a mean velocity of 318 feet per second.

Take, again, the moment when the projectile is leaving the muzzle. The surface exposed is 3040 square inches, and the mean pressure about 5 tons per square inch. There is therefore a mass of matter moving with a mean velocity of 937 feet per second, and with a pressure of 15,200 tons.

Setting aside the effect of velocity, a coefficient of statical friction of 1 in 50 would give a forward strain of 418 tons in the first case, and 304 tons in the second.

388. It has been objected to this view of the question that it involves the existence of shearing resistance in the mixture of gases and liquid, and that such is incredible.

Why is it incredible? Is there not a continuous gradation of properties in various forms of matter? Why should there be a breach of the law of continuity in this case? The products of combustion are a mass of material particles, so are solids. The difference is, that the relations of cohesion and adhesion of the particles *inter se* vary very much in degree.

Gases are reduced to liquids by cooling and mechanical force. Liquids are changed to solids by the abstraction of heat, and the point of liquefaction is altered by pressure. Solids by pressure are made to flow like liquids.

Why then should there be no possibility of a shearing resistance in the products of combustion?—differing, of course, greatly in degree from that in solids, but yet quite sufficient under the circumstances of interior pressure in a gun to make the coefficient of friction of very appreciable magnitude.

389. Another objection has been taken on the ground that in a long gun the amount of this force might equal, or possibly exceed, the backward force in the breech, and that therefore the gun should not either recoil at all, or recoil

forwards. Either case is quite possible, and, in fact, observations tend to show that in some cases the recoil is actually prevented until the projectile leaves the muzzle.

390. In an interesting paper by Mr. H. J. Butter, of the Royal Gun Factory, Carriage Department, read at the Institution of Civil Engineers, 22nd November, 1881, it is stated that "From results obtained by instantaneous photography in connection with the firing of a 25-ton gun, using R.L.G. powder, it was shown that the shot was just clear of the muzzle before the gun moved. More recently it was ascertained by electricity that, in the case of the 6-inch B.L. gun using pebble powder, the shot was within 2 inches of quitting the muzzle when the first movement of the gun occurred." *

391. Now, if there were no opposing force, there could be no doubt that the gun would begin to move very nearly at the same instant as the shot, and setting aside the friction of the carriage in the slide, on the one hand, and the friction of the projectile on the other, the velocity of the recoil would be to that of the shot, inversely as the respective masses moved; and if these be, for example, as 100 to 1, and the muzzle velocity 1800 feet per second, the gun must have acquired a velocity of 18 feet per second backwards when the projectile left the muzzle. But we are informed that in reality it had not moved at all.

392. Again, it is stated by Major Mackinlay, R.A., in his 'Text-book of Gunnery,' 1887, that "by a French experiment it was found that a 24-cm. (9·4-inch) gun had only recoiled 1·180 inch during the time that it had taken for the projectile to travel all the way down the bore; the velocity of recoil was thus 12·86 feet, and it attained its maximum velocity of 17 feet at a period 0·048 second later." In this case the gun was a comparatively short one, with a moderate charge of powder, so that the effect of friction in producing the immediate recoil was not so great.

* Mr. Butter's statement must be taken with reserve. It is quite possible that there are cases in which the gun does not move backwards until the projectile leaves the muzzle, but that is not always the case.

393. Mr. William Anderson observed that in the case of a 10-inch gun fired from a Moncrieff disappearing carriage the gun did not move at all till the projectile left the muzzle.

394. There is therefore, I think, very strong presumptive evidence of the existence of important longitudinal forward strains between the trunnions and the muzzle, the magnitude and perhaps the existence of which have not hitherto been suspected.

395. I now proceed to the investigation of these strains as follows:—

1. *Strains due to Inertia of Mass in front of Trunnions.*

396. Let M = total mass of gun and recoiling part of carriage;

m = mass of the portion of the gun in front of any section at x;

R = outer radius of chase at x;

ρ = radius of bore;

P = maximum internal powder pressure.

Then the moving force is $P\pi\rho^2 = M\frac{dv}{dt}$, and the moving force acting at the section at x will be

$$\frac{m}{M} \cdot M\frac{dv}{dt} = \frac{m}{M} \cdot P\pi\rho^2,$$

and taking the weights W and W_1, and observing that the sectional area at $x = \pi(R^2 - \rho^2)$, we get

$$\text{Strain per sq. inch at } x = \frac{W_1}{W}\,\frac{P\rho^2}{(R - \rho^2)}.$$

2. *Forward Strain due to the Differences between the Area of the Obturation and that of the Bore.*

397. This is equal to the maximum pressure multiplied by the difference of area, and it may be taken up at the trunnions, in which case it only acts as easing the recoil.

3. *Forward Strains due to the Projectile.*

398. This strain is threefold. First, there is the strain required to force the rotating ring into the grooves. Its amount depends, of course, upon the dimensions and material of the rotating ring, but it is probably of no great amount, and only acts for a short time at the beginning of the motion of the projectile, retarding its motion and increasing the maximum pressure. Its effect may therefore be considered as included in that of the pressure.

The next part of the strain is that due to friction. The friction of the projectile on the chase considered as a sliding friction is simply the weight of the projectile multiplied by the coefficient of friction, and is so small that it may be neglected. There is, however, the friction arising from the reaction of the rifle grooves against the projectile, which may be estimated by the well-known formula.

Strain arising from Friction of Products of Combustion.

399. There are two hypotheses by which this subject may be investigated.

First, that the resistance due to friction varies directly as the density and as the square of the velocity of the products.

Second, that it is independent on the velocity and varies directly as the pressure.

1*st Hypothesis.*

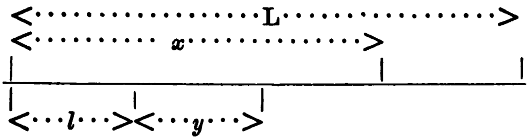
400. Let it be assumed that

$$R = f \frac{\delta s v^2}{2g}, \tag{1}$$

the expressions given by Rankine and others, where

f is a numerical coefficient;
δ the density or weight per cubic foot of the powder;
s the surface exposed;
v the velocity.

Unities—feet, tons, seconds.



Let L = total length of bore.
l = equivalent length of chamber.
x = distance travelled by projectile at any time t.
y = any intermediate length less than x.
ρ = radius of bore.
C = capacity of chamber.
w = weight of charge.
v = velocity of projectile at x.
V = muzzle velocity.

For the sake of simplicity I assume that the gravimetric density of the charge = 1, and that the powder is a quick powder, to which Sarrau's monomial formula is applicable.

Therefore $v \propto x^{\frac{3}{16}}$, and

$$v = \mathrm{V}\left(\frac{x - l}{\mathrm{L} - l}\right)^{\frac{3}{16}}. \tag{2}$$

Assuming that the velocity of the products of combustion increases uniformly from zero at the breech to v at the projectile, we get the velocity at y

$$v_{\prime} = \mathrm{V}\left(\frac{x - l}{\mathrm{L} - l}\right)^{\frac{3}{16}} \cdot \frac{y + l}{x}, \tag{3}$$

which is the value of v to be used in (1).

Now the density at

$$y = \frac{w}{C} \cdot \frac{l}{l + y}$$

and the surface is $2\,\pi\,\rho\,d\,y$, consequently

$$d\,R = \frac{f\,w}{C} \cdot \frac{l}{l + y} \cdot \frac{2\,\pi\,\rho\,d\,y}{2\,g}\,V^2 \left(\frac{x - l}{L - l}\right)^{\frac{3}{8}} \left(\frac{y + l}{x}\right)^2 \quad (4)$$

and writing

$$B \text{ for } \frac{\pi\,\rho\,l\,w\,V^2}{C\,.\,g}$$

$$R = f\,B\,\frac{1}{(L - l)^{\frac{3}{8}}}\,\frac{(x - l)^{\frac{3}{8}}}{x^2} \int_0^x (y + l)\,d\,y \qquad (5)$$

$$= f\,B\,\frac{1}{(L - l)^{\frac{3}{8}}}\,\frac{(x - l^{\frac{3}{8}})}{x^2} \left\{\frac{y^2}{2} + l\,y + \text{Constant}\right\} (6)$$

When $y = 0$ the resistance is that due to the surface of the length l, but then the velocity is zero, therefore $R = 0$, and Constant $= 0$, and when $y = x$

$$R = f\,B\,\frac{1}{(L - l)^{\frac{3}{8}}} \cdot (x - l)^{\frac{3}{8}} \left\{ \tfrac{1}{2} + \frac{l}{x} \right\},$$

and when $x = L$

$$R = f\,B \left\{ \tfrac{1}{2} + \frac{l}{L} \right\}. \qquad (7)$$

Second Hypothesis.

401. Adopting the same notation and making use of Noble and Abel's formula, if p_x be the pressure on the base of the projectile at any point x

$$p_x = P \left\{ \frac{\cdot 43\,l}{x - \cdot 57\,l} \right\}^{1 \cdot 237}; \qquad (1)$$

but the pressure at the breech is always greater than that on

the projectile, therefore let it be denoted by $a\,p_x$, then the mean pressure

$$= p x \frac{1 + a}{2}. \qquad (2)$$

The area of surface exposed $= 2\pi\rho x$, and if ϕ be the coefficient of friction,

$$R = 2\pi\rho x p_x \frac{1 + a}{2} \cdot \phi. \qquad (3)$$

Now

$$p x = P\left(\frac{\cdot 43\, l}{x - \cdot 57\, l}\right)^{1\cdot 237}.$$

Now P is a maximum for some value of $x = x_1$ greater than l which will be found by the relation

$$P_1 = P\left(\frac{\cdot 43\, l}{x_1 - \cdot 57\, l}\right)^{1\cdot 237}.$$

Where P_1 is the maximum powder pressure in the gun, and P the pressure in a close vessel $= 43$ tons, from which we get

$$x_1 = \left(\frac{P}{P_1}\right)^{\cdot 808} \cdot 43\, l + \cdot 57\, l, \qquad (4)$$

introducing which value of x into (3) we get the maximum resistance

$$R = 2\pi\rho\phi p_x \cdot \frac{1 + a}{2} \cdot \left\{\left(\frac{P}{P_1}\right)^{\cdot 808} \cdot 43\, l + \cdot 57\, l\right\}. \qquad (5)$$

402. *Application to 6-inch Gun.*

1st Hypothesis.

$L = 13\cdot 33$ feet.
$l = 2\cdot 833$ „
$\rho = 0\cdot 25$ „
$C = 0\cdot 551$ cubic feet.
$w = 34$ lbs. $= \frac{34}{2240}$ tons,

from which

$$B = 6656$$

and by formula (7)

$$R = 4746\, f.$$

403. Now, if we take the rupturing force as 446 tons, as shown in § 382, we have

(*a*) Strain arising from inertia of the tube and coil, as follows:—

$$= \frac{W_1}{W} P \pi \rho^2$$

$$W_1 = 2 \cdot 27 \text{ tons.}$$
$$W = 6 \cdot 5 \quad \text{,,}$$
$$P = 25 \text{ tons per sq. inch.}$$
$$\pi \rho^2 = 28 \text{ sq. inches.}$$

Therefore,

$$\text{Strain} = \frac{2 \cdot 27}{6 \cdot 5} \times 25 \times 28 = 244 \text{ tons.}$$

404. (*b*) Strains arising from difference of area ofobturator and bore

$$= 1 \cdot 52 + 25 \text{ tons} = 38 \text{ tons.}$$

405. (*c*) Forward strain due to projectile.

In this case the projectile had no rotating band, the rotation being given by ribs, and the twist was a uniform twist of 1 in 30.

The coefficient of friction n is taken as $\frac{1}{5}$th, and the work done to overcome friction would be

$$\frac{1 \cdot 737 \rho^2 P l^{1 \cdot 237}}{\cdot 237 \, m n} \cdot \left\{ \frac{1}{(\cdot 43 \, l)^{\cdot 237}} - \frac{1}{(L + \cdot 43 \, l)^{\cdot 237}} \right\}$$

and since

$$\rho = 0 \cdot 25, \quad P = 25 \text{ tons per sq. inch,} \quad l = 2 \cdot 833,$$
$$L = 10 \cdot 5, \quad m = 30, \quad n = \tfrac{1}{5},$$

we get

$$\text{Work done} = 15 \cdot 79 \text{ foot-tons}$$

and

$$\text{Mean force} = \frac{15 \cdot 79}{10 \cdot 5} = 1 \cdot 504 \text{ tons}$$

and

$$\text{Maximum force} = \frac{\pi \rho^2 P}{2 m} = 11 \cdot 66 \text{ tons;}$$

therefore, since $n = \frac{1}{5}$

$$\text{Maximum forward force} = \frac{11 \cdot 66}{5} = 2 \cdot 332 \text{ tons};$$

also

$$\text{Sliding friction} = \frac{100}{5 + 2240} - \cdot 009 \quad ,,$$

$$\text{Total force to overcome friction} = 2 \cdot 341 \quad ,,$$

406. Thus a, b, c together amount to 284 tons, and as the rupturing strain was 446 tons, there remains 162 tons to be overcome by the friction of the products.

Therefore

$$R = 4746 f > 162$$

or

$$f > \frac{162}{4746} > \cdot 0341$$

407. The coefficient f given by Rankine for ordinary gases is $\cdot 006$, therefore it does not seem at all improbable that, for the mixed products of combustion, it may be quite sufficient to rupture the tube.

Second Hypothesis.

408. The value of x_1 is found to be $3 \cdot 508$ feet, and taking $a = 1 \cdot 25$, we get

$$px \frac{1 + a}{2} = 4050 \text{ tons per sq. foot.}$$

Therefore from (5)

$$R = 22310 \, \phi,$$

and therefore making $R = 162$ tons

$$\text{we get } \phi > \frac{162}{22310}$$

or

$$\phi > \cdot 00726.$$

409. The value of ϕ is, of course, different from that of f obtained by the first hypothesis, the first being the ordinary

coefficient of friction as depending only on pressure, the second being a coefficient which varies with the density and velocity of the products.

410. Further confirmation of the longitudinal strain due to friction of the products is given by the fact that in a 6-inch wire gun, made from my designs at Aboukoff, by Admiral Kolokoltzoff, the part of the steel tube in front of the trunnions, about 8 feet in length, was actually elongated $\frac{3}{16}$ of an inch after firing 500 rounds.

This gun differed from the gun which was fired at Woolwich, inasmuch as the greatest part of the forward strain was taken up by the jacket a little in front of the trunnions, where the sectional area of the jacket (also cast iron) was very much greater than that at the muzzle of the Woolwich gun. The eight feet of the tube in advance of this was quite free to elongate, and there was no force acting on it except the friction of the projectile and that of the products of combustion.

It could not be elongated by any deformation arising from the internal pressure, as the pressure on that part was very far below the elastic limit for compression.

411. The foregoing observations and investigations must be taken with reserve, but I think they afford strong presumptive evidence that this hitherto neglected effect of the friction of the products has a very important bearing on the longitudinal strain on the chase of a gun in front of the trunnions.

The accident to the 6-inch gun referred to in (§ 380), was certainly due to a forward force which I am unable to account for otherwise, and I think it not only unsatisfactory, but very unfair, that I was neither allowed to give my views to the Ordnance Committee, nor to see the report which they made to the War Office. To treat this report as "confidential" as between myself and the Ordnance Department is not only absurd, but it is unbusiness-like and unfair, and it is difficult to understand how such reserve can be beneficial to the public service.

Chambering.

412. A good deal of importance has been attached to chambering, as if it were a means of reducing the maximum pressure in a gun, without affecting its initial velocity. This it cannot do. The maximum pressure, *cæteris paribus,* varies as the gravimetric density, which has nothing to do with the diameter of the chamber. The only effect of increasing the diameter of the chamber is to decrease its length for the same charge of powder, and of course this decreases the total length of the gun, but, as may easily be shown, does not decrease its weight. In fact it somewhat increases it, owing to the extra size of the breech plug. The muzzle velocity, *cæteris paribus,* depends on the length of travel of the projectile, so that the actual length of the gun is decreased simply by the difference between the length of the chamber and its equivalent length. This decrease of length is doubtless an advantage for naval guns.

413. There is no difficulty in constructing chambered wire guns, and I am of opinion that a 12-inch chambered wire gun of about 65 tons in weight and 30 feet long, might be made with equal power of penetration as the 110-ton 16·25 inch Elswick gun at 1000 yards, and which would exceed it at any longer distance, working, of course, at a higher pressure, but with no greater relative strain on the gun.

414. As regards the alleged wave pressure, I will only repeat what I have shown before when treating of Ignition, that with a properly constructed cartridge, so as to insure rapid Ignition, there will be no wave pressure.

415. The unchambered gun has however the advantage that any weight of charge can be fired with full gravimetric density, while in the chambered gun any charge below the full charge must be fired with lower gravimetric density, depending on its weight, and consequently with a loss of ballistic effect.

Rifling.

416. The system of rifling has little or no effect on the muzzle velocity of the projectile or the maximum powder pressure. As regards accuracy of fire, it matters nothing whether the twist be uniform or increasing, provided the increasing twist ceases to increase a short distance from the muzzle. If in the last three or four calibres of length the projectile has the right amount of rotation and is truly centred, which it may be with either system of rifling, it matters nothing, so far as accuracy of fire is concerned, what went before, or how that rotation was acquired.

The danger of over-riding the grooves, or of any partial jamming, is greatest with the uniform twist at the beginning of the motion and decreases very rapidly as the projectile acquires velocity, whilst with the increasing twist it is exactly the reverse, consequently the effect of any such partial jamming, depending as it does on the sudden loss of energy, is much greater with the increasing twist.

Erosion.

417. I will conclude this chapter by a few words on this subject. I have already given my reasons, when dealing with the powder question, for thinking that the very serious increase of erosion is due chiefly to the enormous increase in the volume of the products of combustion, consequent on the increased charges required by low pressure powder and to the very high temperature of combustion of the brown powder. The Russian 6-inch wire gun above mentioned fired 1000 rounds with $39\frac{1}{2}$ lbs. of black prismatic powder, and the erosion was moderate, and the gun still serviceable, although of course the accuracy of fire was diminished. General

Maitland has given the following formula for the life of a gun firing brown prismatic powder—

$$\text{Number of Rounds} = \frac{2400}{\text{calibre in inches}} - 50.$$

Which for a 6-inch gun gives 350 rounds. With black prismatic powder the Russian gun fired 1000 rounds, and was still serviceable. Its muzzle velocity with $39\frac{1}{2}$ lbs. charge and 122 lbs. projectile was 1715 feet per second, and the maximum powder pressure $24\frac{1}{2}$ tons.

418. To meet the rapid erosion of the Woolwich steel guns a system of lining has recently been adopted. A thin liner of steel has been inserted, extending throughout the chamber and about a third of the chase, with the idea of being replaced by a new liner when the erosion becomes too great, but, as might have been foreseen, these liners are apt to twist round by the reaction of the projectile, and thus, after a few rounds, the grooves of the liners do not correspond with those of the front part of the chase. To meet this difficulty various devices have been tried, but with little or no success; and this system of partial lining may be pronounced a failure. I cannot but look upon this as a most dangerous state of things, and one which must lead to the abandonment of this system of liners. If liners are used at all they ought to extend throughout the whole travel of the projectile.

419. It is hardly necessary to say that this chapter is not a treatise on gun construction. My aim in it has been to show the relation that Internal Ballistics have to that subject, to point out the nature, origin, and extent of the principal strains to which a gun is exposed, and to emphasise the great importance of the separation of the bursting and the longitudinal strains.

I have not entered upon the question of the comparative merits of hooped and wire-bound guns, either as regards cost, time required for construction, or safety. In a previous work I have treated of the general principles of the application of wire, and I will only add that actual experience may now be

appealed to, to show the great superiority of the wire system to that of hoops. I wish, however, to call attention to the remarkable work of General Kalakoutski on the internal strains inherent in cast-steel forgings, and the dangerous character they may assume under the ordinary practice of hardening, tempering, and annealing, and the aggravation of the danger which may arise from the minute inaccuracies of workmanship which it is impossible to avoid in such a complicated structure as a large hooped gun. From all these difficulties the wire system of construction is free, and I feel confident that the views on this subject which for the last thirty years I have been advocating, will ere long be admitted to be correct even by those who have so long looked upon them as the dreams of a visionary theorist.

CHAPTER V.

GUNS CONSIDERED AS THERMODYNAMIC MACHINES.

420. Count St. Robert in his 'Principes de Thermodynamique,' Turin, 1870, indicated generally the relation of ballistic effect to thermodynamic laws, and in a paper presented to the Institution of Civil Engineers, and published in vol. lxxx., Session 1884–85, of their 'Minutes of Proceedings,' I endeavoured to apply Count de St. Robert's method to determine the initial velocity and velocity of recoil of rifled guns.

421. The subject is one of considerable interest, and as the paper above mentioned contained many typographical and other errors, I have modified it in the present chapter.

422. A gun is a machine for the conversion of heat into mechanical force, just as much as is a steam-engine. In the gun the heat operates through the mechanical force of expansion of the gases, which are evolved simultaneously with the heat.

These gases pass through a cycle of which the initial state is the high temperature of ignition of the powder, and the final state that when the projectile leaves the gun. Consequently the heat expended must be represented by the work done in overcoming the various resistances, and in imparting energy to the projectile, the gun, and the products of combustion.

423. Let

Δ H be the units of heat abstracted from the products of combustion in passing from the initial to the final temperature.

ΔQ, the units of heat passing from the products into the body of the gun.

ΔI, the increment of internal work in the gases during the same time.

ΔW, the external work done, consisting of

(*a*) The statical resistance of the air to the motion of the projectile, in other words, the atmospheric pressure, not including the increased resistance due to velocity.

(*b*) The work done in giving rotation to the projectile.

(*c*) The friction of the projectile on the rifling.

(*d*) The work done in overcoming the friction of the gas-check and projectile.

(*e*) The work done in overcoming the friction of the products of combustion in the bore.

(*f*) Work done in stretching the gun circumferentially and longitudinally.

ΔV, the sum of the energy acquired by the whole system of projectile, gun, gun-carriage, products of combustion, and $\int R\,ds$ the resistance of the air due to the velocity of the projectile.

J, Joule's coefficient of the mechanical equivalent of heat, or

$$772 \text{ foot-lbs.} = 1 \text{ unit.}$$

424. Then we have the following relation,

$$J\Delta H = J\Delta Q + \Delta I + \Delta W + \tfrac{1}{2}\Delta V. \qquad (1)$$

Determination of the above Functions.

Δ H.

425. According to Noble and Abel's researches the products of combustion of gunpowder are

43 per cent. gaseous	.. ..	spec. heat	0·186
57 per cent. non-gaseous	..	do.	0·450

If, therefore, w be the weight of powder burnt, t_0 and t the initial and final temperatures, we get

$$\begin{aligned} \Delta \mathrm{H} &= \{\cdot 57\, w \times \cdot 450 + \cdot 43\, w \times \cdot 086\}\,(t_0 - t) \\ &= \cdot 3365\, w\,(t_0 - t). \end{aligned} \qquad (2)$$

Now t_0, the initial temperature of combustion, is from 2274° C. to 2374° C. absolute. (In what follows the value taken is 2342° C., or 4215° F. absolute.)

t is obtained from Noble and Abel's equation

$$t = t_0 \left(\frac{v_0\,(1 - a)}{v - a\,v_0}\right)^{\frac{C_p - C^v}{C_v + \beta\lambda}} = t_0 \left(\frac{1 - a}{\frac{v}{v_0} - a}\right)^{\frac{C_p - C_v}{C_v + \beta\lambda}}$$

where

$$\begin{aligned} a &= \cdot 57. \\ C_p &= \cdot 2324 = \text{sp. heat at constant pressure.} \\ C_0 &= \cdot 1762 = \text{do. at constant volume.} \\ B &= 1\cdot 2957 \qquad \lambda = 045 \end{aligned}$$

β

Therefore

$$t = t_0 \left(\frac{\cdot 43}{\frac{v}{v_0} - \cdot 57}\right)^{0\cdot 074} \qquad (3)$$

v_0 and v being the volumes before and after expansion.

Δ Q.

426. This denotes the heat abstracted by the gun. About this there is much uncertainty.

Count St. Robert, from experiments made with small arms,

estimated that about 250 units of heat were imparted from one kilogramme of powder, or about 30 per cent.

Noble and Abel, from experiments made on a 12-pounder 3-inch gun, found that 545 units of heat were abstracted in 9 rounds of $1\frac{3}{4}$ lbs. each, or $7 \cdot 2$ kilogrammes, which gives $60\frac{1}{2}$ units per round, or $75\frac{1}{2}$ units per kilogramme of powder, or about $9 \cdot 6$ per cent.

They further estimated that with a 10-inch gun the loss would not exceed $3\frac{1}{2}$ per cent.

427. There can be no doubt that the percentage of loss is much less in large guns, because whilst the absorbing surface only increases as the square of the lineal dimensions in similar guns similarly loaded, the weight of charge, and consequently the quantity of heat evolved, increases as the cube.

Without pretending to any great accuracy in a question where the many elements, or some of them, are uncertain, an approximate solution of the problem may be attempted.

428. In (§ 103), it was shown that if z be the loss of temperature,

$$\log z + z \log a = \log \frac{\mathrm{H}\, e\, e' \tau a^{\mathrm{T}_0}}{c} \log \frac{\sigma}{w}; \qquad (4)$$

where $a = 1 \cdot 0077$, a constant;

$\mathrm{H} = \cdot 000237$ „

e = emissive power of gases, assumed $= 1$;

e' = absorbing power of metal $= \cdot 15$;

c = mean specific heat of products of combustion $= \cdot 186$;

T_0 = mean temperature of products;

σ = surface exposed in dm.^{2}

w = weight of charge in kilogrammes.

With these values (4) becomes

$$\log z + \frac{z}{300} = \log \{\cdot 0001910 \times 1 \cdot 0077^{\mathrm{T}_0} \times \tau\} + \log \frac{\sigma}{w} \quad (5)$$

from which z is easily determined.

Then $z \times c$ = units of heat represented by the fall of

temperature, and if the total units evolved from 1 kilogramme be taken at 728,

$$\frac{z\,c}{728} = \text{percentage of loss.}$$

429. The above expression (5) contains τ and T_0. The former is the time during which the gun is exposed to the heated products, whilst the projectile is passing along the bore. This may be estimated, if we know the length of the gun and the muzzle velocity, by means of M. Sarrau's formula.

In the case of a quick powder this formula is of the form

$$V = C\,l^{\frac{3}{16}}.$$

If, therefore, we represent the velocities by the ordinates of a curve, of which the corresponding abscissæ represent the distance travelled by the projectile, the area of this curve, taken from $l = 0$ to $l = l$ (the length of travel) divided by l, will give the

$$\text{Mean velocity} = \frac{C}{l}\int_0^l x^{\frac{3}{16}}\,d\,x$$

$$= \tfrac{16}{19}\,C\,l^{\frac{3}{16}};$$

and since

$$C = \frac{V}{l^{\frac{3}{16}}}$$

we get

$$\text{Mean velocity} = \cdot 8421\,V.$$

Therefore the time which the projectile takes in reaching the muzzle is

$$\frac{l}{\frac{16}{19}\,C\,l^{\frac{3}{16}}} = \frac{19\,l^{\frac{13}{16}}}{16\,c},$$

and since

$$C = \frac{V}{l^{\frac{3}{16}}}$$

we get finally the time in reaching the muzzle

$$= 1 \cdot 188\,.\,\frac{L}{V}. \qquad (6)$$

430. The temperature of the gun varies as the projectile passes along the bore, but it will be sufficient for the present purpose to take T_0 as the mean temperature during the time that the projectile is in the gun. This may be found as follows:—

431. According to Noble and Abel's formula

$$t = t_0 \left(\frac{v_0 (1 - a)}{v - a v_0}\right)^{\frac{C_p - C_v}{C_v + \beta \lambda}}$$

t and t_0 being absolute temperatures, or, as shown before,

$$t = t_0 \left(\frac{\cdot 43}{\frac{v}{v_0} - \cdot 57}\right)^{\cdot 074}.$$

Since the volumes are proportional to the distance passed over, taking for v_0 the equivalent length of the chamber, that is to say, the length of a cylinder whose diameter is that of the bore and whose capacity is that of the chamber, and making this equivalent length the unity of length, and y the distance moved by the shot measured in the same unity, the value of v will be $v_0 + y = 1 + y$, and

$$t = t_0 \left(\frac{\cdot 43}{\frac{1+y}{1} - \cdot 57}\right)^{\cdot 074} = t_0 \left(\frac{\cdot 43}{y + \cdot 43}\right)^{\cdot 074} = \frac{\cdot 9394\, t_0}{(y + \cdot 43)^{\cdot 074}}. \quad (7)$$

432. If then the temperature be represented by a curve, the area of that curve divided by the length of abscissa will give the mean temperature. But the area of the curve is

$$\int_l^L \frac{\cdot 9394\, t_0}{(y + \cdot 43)^{\cdot 074}}\, dy \qquad (8)$$

where L is the length of the gun including the equivalent length of the chamber, and l is the equivalent length of chamber, or if l be taken as unity, L is the number of expansions.

Integrating the above, and dividing by L − 1, we get the mean temperature of the gases.

$$T_0 = 1\cdot014\, t_0 \frac{(L + \cdot43)^{\cdot926} - (l + \cdot43)^{\cdot926}}{L - 1}. \qquad (9)$$

The value of T_0 in (5) is therefore

$$T_0 = 1\cdot014\, t_0 \frac{(L + \cdot43)^{\cdot926} - (1\cdot43)^{\cdot926}}{L - 1}. \qquad (10)$$

433. The formula for loss of temperature is therefore

$$\log z + \frac{z}{300} = \log \{000191 \times \tau \times 1\cdot0077^{T_0}\} + \log \frac{\sigma}{w} \quad (11)$$

in which z is the loss of temperature;

τ the time of travel of shot

$$= 1\cdot188 \frac{l}{V}; \qquad (12)$$

T_0 the mean absolute temperature of the gases

$$= 1\cdot014\, t_0 \frac{(L + \cdot43)^{\cdot926} - (1\cdot43)^{\cdot926}}{L - 1};$$

L = length of travel of shot + equivalent length of chamber divided by equivalent length of chamber;

t_0 = temperature of combustion = 2274° C.;

σ = surface exposed in dm²;

w = weight of charge in kilogrammes.

434. If the value of τ from (12) be introduced into (11), it becomes

$$\log z + \frac{z}{300} = \log \left\{\cdot00226 \frac{l}{V} \times 1\cdot0077^{T_0}\right\} + \log \frac{\sigma}{w}. \quad (13)$$

435. The above formula must be taken with reserve, as it represents the loss of heat on the hypothesis that the combustion of the powder is uniform throughout the whole time,

and it is, moreover, liable to uncertainty with respect to the value of the emissive power of the products of combustion. It is probable, however, that in the case of large guns, with heavy charges of slow-burning powder, the approximation is tolerably correct.

Δ I. *Internal Work of Gases.*

436. The internal work in a perfect gas expanding is zero, and as powder gases approach very closely to the condition of a perfect gas, we may assume $\Delta . \mathrm{I} = 0$.

Δ W. *External Work.*

437. (*a*) The first item of external work is the work done against the atmospheric pressure, apart from any increase of resistance due to the velocity of expulsion.

If therefore p = atmospheric pressure;
A = area of bore;
L = length of travel of shot;
R = resistance;

the work done $= \mathrm{R\,L} = p . \mathrm{A} . \mathrm{L}$; (13)

Work done in giving Rotation to the Shot.

438. (*b*) Let W = weight of shot;
m = number of calibres to one turn of the shot or twist = 1 in m;
V = muzzle velocity of shot;
$\cdot 707$ = radius of gyration for cylindrical body revolving round its axis.

Then the velocity of rotation of the centre of gyration is $\dfrac{\cdot 707\,\pi \mathrm{V}}{m}$ and the mass is $\dfrac{\mathrm{W}}{g}$;

therefore

$$\text{Work done} = \frac{\mathrm{W}}{2g}\left(\frac{\cdot 707\,\pi \mathrm{V}}{m}\right)^2. \qquad (14)$$

Friction of the Projectile in the Groove.

439. (*c*) For the sake of simplicity, the twist is assumed to be uniform. Then if the pitch of the rifling be 1 in m and P the pressure on the base of the shot

$$\text{Force to give rotation} = \text{P} \cdot \frac{\pi}{2\,m}. \qquad (15)$$

If p be the powder pressure at any point x of the travel of the shot, the pressure on the base will be $p\,\pi\,\rho^2$, ρ being the radius of the base, and substituting in (15), we get

$$\text{Force to give rotation at } x = \frac{\pi^2\,\rho^2}{2\,m} \cdot p. \qquad (16)$$

If P_1 be the initial pressure

$$p = \text{P}_1 \left\{ \frac{\cdot 43}{\frac{v_1}{v_0} - \cdot 57} \right\}^{1\cdot 237}$$

and force to give rotation at x

$$= \frac{(\pi\,\rho)^2}{2\,m} \cdot \text{P}_1 \left(\frac{\cdot 43}{\frac{v_1}{v_0} - \cdot 57} \right)^{1\cdot 237}$$

and if $\frac{1}{n}$ be the coefficient of friction, and the pressure be in tons per square inch, this reduces to

$$\frac{1}{n} \cdot \frac{\rho^2}{m} \cdot \frac{1\cdot 737\ \text{P}_1}{\left(\frac{v_1}{v_0} - \cdot 57 \right)^{1\cdot 237}}. \qquad (17)$$

Now if l be the equivalent length of the chamber in feet, and a the area of the bore

$$v_0 = a\,l, \quad v = a\,(x + l), \quad \frac{v_1}{v_0} = \frac{x + l}{l}.$$

and we get

$$\text{Work done} = \frac{\rho^2}{n\,m} \times 1\cdot 737\ \text{P}_1 \int_0^l \frac{d\,x}{\left(\frac{x + l}{l} - \cdot 57 \right)^{1\cdot 237}} \qquad (18)$$

in foot-tons.

Work done in overcoming the Grip of the Rotating Ring when used, or the Gas Check and the Friction of the Shot.

440. (*d*) The friction proper of the shot, as distinct from the friction due to the reaction of the rifled grooves, is only $\frac{W}{n}$ when W is the weight of the shot, so that the work done is $\frac{W}{n}.L$ which is so small that it may be neglected. The force required to press the shot into the grooves is only at the beginning of the motion, and is only a small fraction of the powder pressure, and as this ceases as soon as the ring has entered the grooves, the work done is quite insignificant, and may also be neglected.

Work done in overcoming the Friction of the Products of Combustion in the Bore.

441. In a previous chapter the question of the effect of the friction of the products was discussed on two hypotheses, with reference to the longitudinal strain produced on the chase of the gun.

In now attempting to ascertain the work done in overcoming this friction I will adopt the second hypothesis (§ 401), which supposes that the friction is proportional to the pressure.

442. When the projectile is at any point x the resistance due to the friction is by (3) (§ 401)

$$R = 2\,\pi\,\rho\,x\,p_s\,\frac{(1+a)}{2}\,\phi\,, \qquad (19)$$

ϕ being the coefficient of friction.

Now the front portion of this mass of products moves forward $d\,x$, whilst the rear portion is at rest. Wherefore

the mean motion is $\frac{dx}{2}$, and the element of work done is therefore

$$dW = 2\pi\rho x \frac{p_x(1+a)}{2}\phi \cdot \frac{dx}{2}; \tag{20}$$

but

$$p_x = P\left(\frac{\cdot 43\, l}{x - \cdot 57\, l}\right)^{1\cdot 237}; \tag{21}$$

therefore

$$dW = \pi\rho p_x \left(\frac{1+a}{2}\right)\phi\, x\, dx;$$

and substituting for p_x from (21)

$$W = \pi\rho P(\cdot 43\, l)^{1\cdot 237}\left(\frac{1+a}{2}\right)\phi \int_l^L \frac{x\, dx}{(x - \cdot 57\, l)^{1\cdot 237}}$$

or

$$W = \pi\rho P(\cdot 43\, l)^{1\cdot 237}\left(\frac{1+a}{2}\right)\phi \left\{\frac{(L - \cdot 57\, l)^{\cdot 763} - (\cdot 43\, l)^{\cdot 763}}{\cdot 763}\right\}$$
$$+ \frac{\cdot 57}{\cdot 237} l \left\{\frac{1}{(\cdot 43\, l)^{\cdot 237}} - \frac{1}{(L - \cdot 57\, l)^{\cdot 237}}\right\}. \tag{22}$$

Work done in Stretching Gun.

443. (f) The pressure of the gases inside the gun acts upon any elementary shell by expanding it circumferentially and compressing it radially.

Let there be such a shell at radius $= y$, whose breadth is β, and thickness dy.

Let t_y = tension per square inch at y;
f_y = radial compressive force at y;
x = extension under t^y;
l = length $= 2\pi y$;
ϵ = modulus of elasticity.

Then $x = l\frac{t}{E}$.

For any other extension z, less than x, let ϕ be the force exerted, then $z = l\,\frac{\phi}{E}$ or $\phi = \frac{E}{l}\,z$, and the work done through dx is $= \frac{E}{l}\,z\,dz \times \beta\,dy$.

Integrating in respect of z, when $z = x = l\frac{t^y}{E}$, we get

$$\text{Work done} = \int \frac{l\,\beta\,t_y^2}{2\,E}\cdot dy.$$

Replacing l by $2\,\pi\,y$, we get

$$\text{Work done} = \int \frac{\pi\,\beta\,t_y^2\,y\,dy}{E}. \qquad (23)$$

444. Now t_y is a function of y, and if f_1 be the internal powder pressure, ρ and R the internal and external radii, and $m = \frac{R}{\rho}$,

$$t_y = \frac{f_1}{m^2 - 1}\cdot\frac{R^2 + y^2}{y^2};$$

substituting which in (23) we get

$$\text{Work done} = \frac{\pi\,\beta}{E}\cdot\frac{f_1^2}{(m^2 - 1)^2}\cdot\int\left(\frac{R^2 + y^2}{y^2}\right)^2 y\,dy, \qquad (24)$$

which gives by integration

$$\text{Work done} = \frac{\pi\,\beta}{E}\,\frac{f_1^2}{(m^2 - 1)^2}\left\{\frac{m^2 - 1}{2}\,R^2 + 2\,R^2\log m + \frac{R^2 + \rho^2}{2}\right\}. \qquad (25)$$

445. Proceeding in like manner for compression we get

$$\text{Work done} = \frac{\pi\,\beta}{E}\,\frac{f_1^2}{(m^2 - 1)^2}\left\{\frac{m^2 - 1}{2}\,R^2 + 2\,R^2\log\frac{1}{m} + \frac{R^2 - \rho^2}{2}\right\}, \qquad (26)$$

and adding these together,

$$\text{Total work done} = \frac{\pi\,\beta}{E}\,f_1^2\,\frac{m^2 + 1}{m^2 - 1}\cdot\rho^2. \qquad (27)$$

446. If the unities be tons and feet, this gives the work done per lineal foot, and since the surface of 1 lineal foot is $2\ \pi\ \rho$, making β unity, the work done per unit of surface is got by dividing the surface by $2\pi\ \rho$, which gives

$$\text{Work done per unit of Surface} = \frac{m^2+1}{m^2-1} \cdot \frac{f_1^{\,2}}{2\,\mathrm{E}} \cdot \rho, \qquad (28)$$

and this is the work done in foot-tons per square foot of surface of chamber.

Work done in Expanding the Chase.

447. As the chase varies in thickness, the value of m is not constant, but the variation is not of such magnitude as seriously to affect the results, and therefore it will be sufficient to assume a mean thickness of the chase, and make use of the value of m belonging thereto.

448. Proceeding thus, the work done per unit of surface may be determined as above, using, instead of the constant pressure f_1, the varying pressures at each point of the chase, which is a function of the length travelled by the shot.

Let L be the total length of travel;

l the equivalent length of the chamber;

x any intermediate length;

P_1 the pressure in the chamber;

p the pressure at x;

then

$$p = \mathrm{P}_1 \left(\frac{\cdot 43}{\frac{x+l}{l} - \cdot 57} \right)^{1\cdot 287}$$

and the work done in $d\,x$ is

$$\frac{\pi}{\mathrm{E}} \cdot \frac{m^2+1}{m^2-1}\,\rho_2 \cdot \mathrm{P}_1^{\,2} \left(\frac{\cdot 43}{\frac{x+l}{l} - \cdot 57} \right)^{2\cdot 474} d\,x,$$

and integrating this, and taking $x = L$, the total work done is

$$W = \frac{\pi}{E} \cdot \frac{m^2+1}{m^2-1} \cdot \rho^2 P_1^2 \frac{\cdot 43\, l^{2 \cdot 474}}{1 \cdot 474} \left\{ \frac{1}{(\cdot 43\, l)^{1 \cdot 474}} - \frac{1}{(L + \cdot 43\, l)^{1 \cdot 474}} \right\}. \quad (30)$$

Work done in Stretching the Gun between Breech and Trunnions.

449. Assuming that the strain is uniformly distributed over the cross section of the gun, this strain per square inch

$$= \frac{P_1 \rho^2 \pi}{(R^1 - \rho^2)\pi} = \frac{P_1}{m^2 - 1} = \phi.$$

Then if l be the total length from breech to trunnions,

$$\text{Total extension} = \phi \frac{l}{E} = \frac{P_1 l}{(m^2 - 1) E}. \quad (31)$$

450. For any intermediate extension y, the force $= E\frac{y}{l}$, and the work done in $dy = E \cdot \frac{y\,dy}{l}$.

Integrating and making $y =$ the total extension, we get work done $= \phi^2 \frac{l}{2E}$ per unit of area; and since the total area $= 2\pi(R^2 - \rho^2)$, and $\phi = \frac{P_1}{m^2 - 1}$, substituting we get

$$\text{Total work done} = \frac{\pi \cdot \rho^2 \cdot l}{(m^2 - 1) E} \cdot P_1^2. \quad (32)$$

Determination of Δ. V.

451. This is made up of the following items:—

(*a*) The vis viva of the projectile $= \frac{W}{g} \cdot u^2$

(*b*) „ of gun and carrriage $= \frac{W_1}{g} u_1^2$

(*c*) „ of products of combustion $= \int u_{''}^2 \, d\mu$

where W = weight of projectile;

W_1 = weight of gun and recoiling part of carriage;

μ = mass of charge $= \frac{w}{g}$;

u, u_1, and $u_{''}$ being the velocities of projectile, gun, and gases respectively.

452. The integral $\int u_{''}^2 \, d_\mu$ must be taken for the whole mass of the products of combustion from the breech to the muzzle, and it depends on the state of the particles and their respective velocities at the moment when the projectile leaves the muzzle.

453. It is assumed—

1st. That the density of the products of combustion is uniform throughout at any given moment.

2nd. That the velocities of the particles increase uniformly from the breech to the muzzle.

3rd. That the particles in contact with the breech and projectile have respectively the same velocities, viz. u_1 and u. This being so, and the motions being in opposite directions, there must be some point where the gases are at rest, and this point divides the whole length in the ratios of u and u_1. If then l be the total length of the chase, the point of rest will be

$$\frac{u_1}{u + u_1} \cdot l \text{ distant from the breech.}$$

$$\frac{u}{u + u_1} \cdot l \text{ distant from the muzzle.}$$

454. Let x be any distance from the point of rest on the muzzle side, and y any distance from the same point on the breech side, then

$$\text{Velocity at } x = u \cdot \frac{x}{u\,l} \div \frac{u\,l}{u+u_1} = \frac{u+u_1}{l} \cdot x. \quad (33)$$

$$\text{Velocity at } y = u \cdot \frac{y}{u\,l} \div \frac{u_1\,l}{u+u_1} = \frac{u+u_1}{l} \cdot y. \quad (34)$$

455. Let δ = density of products of combustion;
A = area of bore.

Now the moments are equal on each side of the point of rest, and these are, on the muzzle side,

$$\frac{\delta\,A}{g} \cdot \frac{u+u_1}{l} \int_0^{\frac{u\,l}{u+u_1}} x\,d\,x = \frac{\delta\,A}{2\,g}\,l\,\frac{u^2}{u+u_1}, \quad (35)$$

and on the breech side

$$\frac{\delta\,A}{2\,g} \cdot l \cdot \frac{u_1^2}{u+u_1}, \quad (36)$$

and as these are in opposite directions, their algebraic sum is

$$\frac{\delta\,A\,l}{2\,g} \cdot \frac{u^2 - u_1^2}{u+u_1} = \frac{\delta\,A\,l}{2\,g} \cdot (u - u_1). \quad (37)$$

Now $\delta\,A\,l = w$, therefore

$$\int u_{\prime\prime}\,\delta\,\mu = \frac{w}{2\,g}(u - u_1). \quad (38)$$

456. For the vis viva in the direction of the muzzle we have

$$\frac{\delta\,A}{g}\left(\frac{u+u_1}{l}\right)^2 \int_0^{\frac{u\,l}{u+u_1}} x^2\,dx, \quad (39)$$

and in the opposite direction

$$\frac{\delta\,A}{g}\left(\frac{u+u_1}{l}\right)^2 \int_0^{\frac{u_1\,l}{u+u_1}} y^2\,dy. \quad (40)$$

the integrals of which are

$$\frac{\delta A l}{3 g} \cdot \frac{u^3}{u + u_1} \quad \text{and} \quad \frac{\delta A l}{3 g} \cdot \frac{u_1^3}{u + u_1}; \tag{41}$$

therefore the total vis viva is

$$\frac{\delta A l}{3 g} \cdot \frac{u^3 + u_1^3}{u + u_1} = \frac{w}{3 g} \cdot (u^2 + u_1^2 - u u_1), \tag{42}$$

which is the value of

$$\int u_{''}^2 \, d \mu.$$

Determination of $\int R \, d s$, *or the Work done in overcoming the resistance of the Air due to Velocity.*

457. Assuming the resistance to be proportional to the cube of the velocity, $R = a u^2$, a being a coefficient determined experimentally.

From experiments made with the Bashforth chronograph the resistance of a 10-inch ogival-headed projectile at 1000 feet per second is 233 lbs., therefore the resistance at any velocity u is

$$233 \left(\frac{u}{1000}\right)^3 = \cdot 000000233 \, u^3,$$

and

$$a = \cdot 000000233.$$

458. Now the velocity of the projectile, if Sarrau's monomial formula be admitted, is $C \, . \, x^{\frac{3}{16}}$, where C is a constant and x the distance travelled by the projectile. Therefore, if u be the muzzle velocity, and l the total travel of the projectile

$$u = C \, l^{\frac{3}{16}} \quad \text{and} \quad C = \frac{u}{l^{\frac{3}{16}}};$$

therefore, the velocity at

$$x = \frac{u}{l^{\frac{3}{16}}} \times x^{\frac{3}{16}} = u\left(\frac{x}{l}\right)^{\frac{3}{16}};$$

consequently

$$R = \cdot 000000233\, u^3 \left(\frac{x}{l}\right)^{\frac{9}{16}},$$

and

$$R\,d\,x = 000000233\, u^3 \int_0^l \left(\frac{x}{l}\right)^{\frac{9}{16}} d\,x,$$

and integrating between

$$x = l \text{ and } x = 0,$$

$$\int R\,d\,x = \cdot 000000233 \times \cdot 64\, l\,.\, u^3$$
$$= \cdot 0000000149\, l\, u^3. \qquad (43)$$

459. This is in lbs. for a 10-inch projectile, and assuming the resistance to be directly as the area, this must be divided by $\frac{78 \cdot 54}{144}$ to give the resistance per square foot and therefore for any calibre c, the resistance will be

$$\frac{\cdot 000000149}{\cdot 554} \cdot \frac{c^2}{4\,\pi} \cdot l\, u^3;$$
$$= \cdot 0000000214\, c^2\, l\, u^3;$$

or dividing by 2240 for foot-tons

$$= \cdot 00000000000956\, c^2\, l\, u^3. \qquad (44)$$

Unities, feet, seconds, and foot-tons.

To determine the Velocities.

460. From the equality of momenta.

$$m_1\, u_1 = m\, u + \int u_{\prime\prime}\, \delta\, \mu\,.$$

$$\text{or} \quad \frac{W_1}{g} \cdot u_1 = \frac{W}{g} \cdot u + \frac{w}{2\,g}(u - u_1);$$

$$\text{or} \quad W_1\, u_1 = W\, u + \frac{w}{2} \cdot (u - u_1),$$

from which u_1 is obtained in terms of u, or

$$u_1 = \frac{W + \frac{w}{2}}{W_1 + \frac{w}{2}} \cdot u. \qquad (45)$$

461. By substituting the values thus obtained in equation (1), an equation is obtained from which u the muzzle velocity is obtained, and from it, by the last equation, the velocity of recoil is known.

462. *Application to* 10-*inch B.L. Woolwich gun.*

W_1 = weight of gun	27 tons.
W = weight of projectile	500 lbs.
P_1 = maximum powder pressure ..	18 tons per sq. inch.
w = weight of charge	300 lbs.
ρ_1 = radius of chamber	0·5833 feet.
l' = length do.	4·5 feet.
l = equivalent length	8·23 feet.
ρ = radius of bore, 5 inches ..	0·4166 feet.
L = travel of projectile	22·5 feet.
v_0 = capacity of chamber	8316 c. inches.
v = total capacity of gun	29522 „
$\frac{v}{v_0} = 3·55.$	
J = Joule's factor	772.

J Δ H.

463. By (2)

$$\Delta H = \cdot 3365\, w\, (t_0 - t).$$

But

$$w = 300 \qquad t_0 = 4215° \text{ Fah. (absolute)},$$

and

$$t = t_0 \left(\frac{\cdot 43}{\frac{v}{v_0} - \cdot 57}\right)^{\cdot 074} = 4215 \times \left(\frac{\cdot 43}{2\cdot 98}\right)^{\cdot 074} = 3652^\circ,$$

therefore

$$\text{Fall of temperature } t_0 - t = 563^\circ,$$

and

$$J \Delta H = \frac{772 \times \cdot 3365 \times 300 \times 563}{2240} = 19{,}580 \text{ foot-tons.}$$

$J \Delta Q$.

464. By (5)

$$\log z + \frac{z}{300} = \log \{\cdot 000191 \times 1\cdot 0077^{T_0} + \tau\} + \log \frac{\sigma}{w} \cdot$$

Where z is the loss of temperature in degrees Centigrade.

τ, the time of combustion of the charge, which is here assumed to be the same as the time of passage of the projectile to the muzzle, which is, of course, a maximum value, and as shown in formula (6),

$$\tau = 1\cdot 188 \frac{L}{v}, \quad \text{L and } v \text{ in metres.}$$

$$L = 22\cdot 5 \text{ feet} = 6\cdot 86 \text{ metres.}$$

$$v \text{ is assumed} = 2000 \text{ feet} = 612\cdot 6 \text{ metres per second.}$$

$$\therefore \tau = \cdot 0133.$$

T_0 = mean absolute temperature of products in degrees Centigrade, which by (10), when t_0 is the absolute temperature of combustion = 2274° C., is

$$= 1\cdot 014\, t_0 \frac{(L + \cdot 43)^{\cdot 926} - 1\cdot 43^{\cdot 926}}{L - 1};$$

or

$$T_0 = 1 \cdot 014 \times 2274 \times \frac{7 \cdot 29^{\cdot 926} - 1 \cdot 43^{\cdot 926}}{5 \cdot 86} = 1896^\circ \text{C}.$$

σ = surface in decimetres square = $700 \cdot 3$.
w = weight of charge in kilogrammes = $136 \cdot 2$.
And making use of these values

$$\log z + \frac{z}{300} = \log \{ \cdot 000191 \times 1 \cdot 0077^{1896} \times \cdot 0133 \} + \log \frac{700 \cdot 3}{136 \cdot 2}$$
$$= \cdot 72488 + \cdot 71110 = 1 \cdot 43590.$$

From which we get $z = 23^\circ$, and if c = mean specific heat $\cdot 186$, and w the weight of products = $136 \cdot 2$ kilog., the total loss of heat is $23 \times 136 \cdot 2 \times \cdot 186 = 582 \cdot 7$ units (French), or in English units, $582 \cdot 7 \times 3 \cdot 968 = 2312$, or in the equivalent of work done

$$J \Delta Q = \frac{2312 \times 772}{2240} = 796 \cdot 8 \text{ foot-tons.}$$

465.

$$\Delta 1 = 0.$$

Δ W.

466. (*a*) *Resistance of Atmospheric Pressure* = p A L.

$$= \frac{14 \cdot 75 \times 78 \cdot 54 \times 22 \cdot 5}{2240} = 11 \cdot 64 \text{ foot-tons.}$$

467. (*b*) *Rotation of Projectile.*

$$\text{Work done} = \frac{W}{2g} \left(\frac{\cdot 707 \, \pi \, u}{m} \right)^2.$$

Here W = 500 lbs.

$m = 30$, or one turn in 30 calibres.

Therefore

$$\text{Work done} = \frac{500}{64\cdot4}\left(\frac{\cdot707 \times 3\cdot1416}{30}\right)^2 u^2 = \cdot04255\, u^2;$$

$$\text{or, in foot-tons} = \cdot000019\, u^2 \text{ foot-tons.}$$

468. (c) *Friction of Projectile.*

Here by (18),

$$\text{Work done} = 1\cdot737\,\frac{\rho^2}{n\,m}\,.\,P_1\int_0^L \frac{d\,x}{\left(\frac{x+l}{l} - \cdot57\right)^{1\cdot237}}$$

$$= 1\cdot737\,\frac{\rho^2}{n\,m}\,P_1\,l^{1\cdot237}\int_0^L \frac{d\,x}{(x + \cdot43\,l)^{1\cdot237}}$$

which integrated between the limits L and 0 gives

$$\frac{1\cdot737\,\rho^2\,P_1\,l^{1\cdot237}}{\cdot237\,m\,n}\left\{\frac{1}{(\cdot43\,l)^{\cdot237}} - \frac{1}{(L + \cdot43\,l)^{\cdot237}}\right\},$$

and

$$\rho = 5 \text{ inches} = \cdot4166 \text{ feet}, \quad m = 30, \quad \frac{1}{n} = \frac{1}{5}$$

$$P_1 = 2592 \text{ tons per sq. foot}, \quad l = 8\cdot232 \text{ feet}, \quad L = 22\cdot5 \text{ feet}.$$

Making use of which values in the above, we get

$$\text{Work done} = 298\cdot4\left\{\frac{1}{1\cdot349} - \frac{1}{2\cdot165}\right\}$$

$$= 295\cdot4\,\{\cdot7413 - \cdot4619\} = 83\cdot34 \text{ foot-tons.}$$

469. (d) *Friction proper of Projectile and Base Ring.*

The friction of the projectile itself is very small and is by (§ 440)

$$= \frac{W}{n}\,.\,L,$$

or

$$\frac{500}{5 \times 2240} \times 22 \cdot 5 = 1 \cdot 005 \text{ foot-tons.}$$

The pressure required to force the rotating ring into the grooves, though considerable in itself, represents very little work done, as it acts only through a very small space at the beginning of the motion, and its effect is simply to increase temporarily the pressure of the gases, but not to absorb energy permanently.

470. (c) *Friction of Products of Combustion.*

By (22)

$$W = \pi \rho P (\cdot 43\, l)^{1 \cdot 237} \frac{(1 + a)}{2} \phi \left\{ \frac{(L - \cdot 57\, l)^{\cdot 763} - (\cdot 43\, l)^{\cdot 763}}{\cdot 763} + \frac{\cdot 57}{\cdot 237} l \left(\frac{1}{(\cdot 43\, l)^{\cdot 237}} - \frac{1}{(L - \cdot 57\, l)^{\cdot 237}} \right) \right\},$$

where

$$\rho = \cdot 4166, \quad P = \cdot 6192, \quad l = 8 \cdot 232, \quad L = 22 \cdot 5,$$
$$a = 1 \cdot 945 \left(\tfrac{300}{800}\right)^{\frac{1}{2}}, \quad (\cdot 43\, l)^{1 \cdot 237} = 4 \cdot 776, \quad (\cdot 43\, l)^{\cdot 237} = 1 \cdot 349,$$
$$(L - \cdot 57\, l)^{\cdot 763} = 17 \cdot 808^{\cdot 763} = 9 \cdot 000, \quad (L - \cdot 57\, l)^{\cdot 237} = 1 \cdot 979,$$
$$\frac{\cdot 57}{\cdot 237} l = 1 \cdot 980, \quad a = 1 \cdot 252, \quad (\cdot 43\, l)^{\cdot 763} = 2 \cdot 623,$$

from which

$$W = 43550\, \phi \times 8 \cdot 729 = 380000\, \phi.$$

In (§ 408) we found a value for $\phi = \cdot 00725$, but this was derived from an experiment with a 6-inch gun, and with exceedingly doubtful data. On the other hypothesis the coefficient was $\cdot 00341$. For our present purpose, and under every reserve, I assume a coefficient of $\cdot 005$, which gives the work done $380000 \times \cdot 005 = 1900$ foot-tons.

Work done in Stretching Gun.

Chamber.

471. By (28) the work done per foot of surface of the chamber is

$$\frac{m^2+1}{m^2-1} \cdot \frac{f_1^2 \rho}{2\,\mathrm{E}},$$

where

$$\frac{m^2+1}{m^2-1} = \frac{\mathrm{R}^2+\rho^2}{\mathrm{R}^2-\rho^2} = \frac{449}{357} = 1\cdot 28,$$

f_1 = internal pressure = 18 tons per sq. inch,

= 2592 tons per sq. foot,

ρ = 7 inches = 0·5333 feet,

and the surface of the chamber

= 16·5 sq. feet.

E = modulus of elasticity

= 13000 tons per sq. inch.

$$\therefore \mathrm{W} = \frac{1\cdot 28 \times 2592 \times 0\cdot 5333 \times 16\cdot 5}{2 \times 13000 \times 144} = 20\cdot 21 \text{ foot-tons.}$$

Stretching the Chase.

472. By (30) work done is

$$\mathrm{W} = \frac{\pi}{\mathrm{E}} \cdot \frac{m^2+1}{m^2-1} \cdot \rho^2 \mathrm{P}_1^2 \cdot \frac{(\cdot 43\,l)^{2\cdot 474}}{1\cdot 474} \left\{ \frac{1}{(\cdot 43\,l)^{1\cdot 474}} - \frac{1}{(\mathrm{L} + \cdot 43\,l)^{1\cdot 474}} \right\}$$

unities, feet, and tons per square foot; and taking a mean value for R the outer radius = ·8333.

$$\rho = \cdot 4166, \qquad \mathrm{P}_1 = 2592, \qquad \mathrm{E} = 13000 \times 144,$$

$$l = 8\cdot 232, \qquad \mathrm{L} = 22\cdot 5,$$

$$\frac{m^2+1}{m^2-1} = \frac{\mathrm{R}^2+\rho^2}{\mathrm{R}^2-\rho^2} = \frac{\cdot 8333^2 + \cdot 4166^2}{\cdot 8333^2 - \cdot 4166^2} = 1\cdot 666.$$

Therefore

$$W = 50 \cdot 75 \left\{ \frac{1}{6 \cdot 467} - \frac{1}{122 \cdot 2} \right\} = 50 \cdot 75 \{ \cdot 1546 - \cdot 0082 \}$$

$$= 7 \cdot 425 \text{ foot-tons.}$$

Stretching between Breech and Trunnions.

473. By (32) work done is

$$W = \frac{\pi (R^2 - \rho^2) l \,.\, P_1}{E (m^2 - 1)^2}.$$

Hence

$R = 20'' = 1 \cdot 666$ ft. $\qquad \rho = 5'' = \cdot 4166$ ft. $\qquad l = 9$ ft.

$m^2 = \dfrac{R^2}{\rho^2} = 16 \qquad R^2 - \rho^2 = 2 \cdot 6015$

$P_1 = 18 \times 144 \qquad E = 13000 \times 144$

$$\therefore W = \frac{3 \cdot 1416 \times 2 \cdot 6015 \times 9 \times 2592^2}{13000 \times 144 \times 15^2} = 1 \cdot 173 \text{ foot-tons.}$$

Determination of Δ V.

474. (1) Vis viva of projectile :—

$$= \frac{W}{g} u^2.$$

$$\text{Work done} = \frac{W}{2\,g} u^2.$$

$$W = 500 \text{ lbs.} = \cdot 2232 \textit{ tons.}$$

$$\therefore \text{ Work done} = \frac{\cdot 2232}{2 \times 32 \cdot 2} u^2 = \cdot 003465\, u^2 \text{ foot-tons.}$$

475. (2) Vis viva of gun and recoiling part of carriage :—

$$= \frac{W_1}{g} u_1^2.$$

Taking the weight of gun and recoiling part of carriage at 36 tons,

$$\text{Work done} = \frac{36}{64 \cdot 4} u_1^2 = \cdot 559\, u_1^2.$$

476. (3) Vis viva of gases, or $\int u_{\prime\prime}^2\, d\mu$:—

By (42) this is

$$\frac{w}{3g}(u^2 + u_1^2 - u\,u_1,$$

$$\text{and work done} = \frac{w}{6g}(u^2 + u_1^2 - u\,u_1).$$

Now

$$w = \frac{300}{2240} = \cdot 1339 \text{ tons.}$$

$$\therefore \text{ Work done} = \frac{\cdot 1339}{6 \times 32 \cdot 2}(u^2 + u_1^2 - u\,u_1)$$

$$= \cdot 000693\,(u^2 + u^2 - u\,u_1) \text{ foot-tons.}$$

Resistance of Air $\int \mathrm{R}\, d\,x$.

477. By (44) this is $\cdot 0000000000956\, c^2\, l\, u^3$.

Hence

$$c = 10'' \text{ inch} = \cdot 8333 \text{ foot.}$$

$$l = 22 \cdot 5.$$

The value of u must be assumed, and in this case we will take it at 2000 feet per second, which gives work done = 11·45 foot-tons.

Total Work done.

478. The total work done is therefore as follows :—

Heating gun, ...		796·8	foot-tons.
Internal work, ...		0·0	„
Statical resistance of air		11·64	„
Force to give rotation	$\cdot 000019\, u^2$		
Friction of projectile rifling ..	83·34	84·35	„
do. pressure	1·01		
Friction of gun (making $f = \cdot 005$)		1900·00	„
Stretching gun—in chamber ..	20·21	28·81	„
chase	7·43		
longitudinal ..	1·17		
On projectile	$\cdot 003465\, u^2$		
On gun and carriage	$\cdot 559\, u_1^2$		
On gases	$\cdot 000693\,(u^2 + u_1^2 - u\,u_1)$		
Resistance of air $\int \mathrm{R}\,dx$		11·45	„

Adding these and equalising to J Δ H, the equivalent of the heat expended, we get

$$19{,}580 = 2833 + \cdot 004177\, u^2 + \cdot 5597\, u_1^2 \times \cdot 000693\, u\, u_1,$$

but as shown above (45),

$$u_1 = \frac{\mathrm{W} + \frac{w}{2}}{\mathrm{W}_1 + \frac{w}{2}}\, u\,,$$

and

$$\mathrm{W} = 500 \text{ lbs.} \qquad w = 300 \qquad \mathrm{W}_1 = 36 \times 2240.$$

$$\therefore\ u_1 = \frac{650}{80790} = \cdot 00805\, u\,,$$

and

$$u_1^2 = \cdot 0000647\, u^2;$$

therefore

$$\cdot 5597\, u_1^2 = \cdot 0000362\, u^2,$$

and

$$\cdot 000693\, u\, u_1 = \cdot 00000551\, u^2,$$

substituting which in the above we get

$$16747 = \cdot 004207\, u^2,$$

and

$$u = 1995 \text{ feet per second,}$$

which is the muzzle velocity; and

$$u_1 = \cdot 00805 + 1995 = 16\cdot 06 \text{ feet per second;}$$

which is the velocity of recoil.

479. From which we get

Work done for rotation	$= \cdot 000019\, u^2 =$	$75\cdot 62$	foot-tons.
„ friction of gun	$= \;.\;.\;.\; =$	$1940\cdot 00$	„
„ on projectile	$= \cdot 003465\, u^2 =$	$14265\cdot 00$	„
„ on gun and carriage	$\cdot 559\, u_1^2 =$	$149\cdot 00$	„
„ on gases	$\cdot 000693\,(u^2 + u_1^2 - u\,u_1) =$	$2743\cdot 00$	„

480. *Summary of Work done.*

Energy of projectile		13796·0	foot-tons,
„ gun and carriage in recoil		149·0	„
„ gases	2743		
Friction of gases	1940		
		4783·0	„
Rotation		75·6	„
Resistance of air (statical) ..	11·64		
„ $R\int dx$ ..	11·45		
		23·1	„
Friction of projectile	1·01		
„ in rifling	83·34		
		84·4	„
Stretching gun		28·8	„
Equivalent of heat imparted to gun ..		796·8	„
Total		19734·7	foot-tons.

481. The equivalent of the heat expended, as shown in (§ 451), was 19,580 foot-tons, so that the difference is 154 foot-tons, or 0·75 per cent.

482. Taking the whole heat developed in the combustion of the powder at 1298·4 units per lb., the equivalent work of 300 lbs. is.. .. .. .. 134,260 foot-tons.
Of this there is accounted for 19,734 „

showing a loss of .. .. 114,526 foot-tons; the whole of which is in the residual heat of the gases as they escape from the gun at an absolute temperature of 3652° Fah.

483. It thus appears, that if we consider the energy imparted to the projectile, the useful effect only $\frac{13795}{134260} = \frac{1}{9 \cdot 73}$ lbs., or about 10·3 per cent. of the power is utilised.

484. Of the work actually done in the gun, the following are the percentages :—

Useful effect on projectile	69·900	per cent.
Effect on recoil	·764	„
„ on the gases	24·240	„
On rotation and friction of projectiles	·810	„
On expulsion of air stretching gun ..	·250	„
Heating the gun	4·036	„
	100·000	„

485. The observed velocity with this gun was about 2100 feet per second, but as the observed velocity is always somewhat greater than the real muzzle velocity, owing to the continued action of the gases, the actual muzzle velocity would be about 2065 feet per second, or about $3\frac{1}{2}$ per cent. above the calculations. This difference is not more than might be expected, since the two important items of loss of heat and friction of gases are subject to considerable uncertainty. It appears, therefore, that the Thermodynamic method gives very approximately correct results.

486. The foundation of it is of course the fall of tempera-

ture in the gun, or more correctly stated, the difference between the temperature of combustion and the temperature of the products at the time the projectile reaches the muzzle, and this is got by using Noble and Abel's equation, where the change of temperature is given as a function of the change of volume.

487. In this equation it is supposed that the whole of the powder is converted into products before the change of volume begins, and that it is then at the highest temperature, and that afterwards in expanding and doing work, it falls to the lower temperature.

488. To this it may be objected, that in the case of a gun, these conditions do not exist; that the powder begins to do work as soon as the combustion begins; that it also begins by imparting some part of its heat to the gun, consequently lowering the temperature and pressure of the products, and it may be said, that under such a simultaneous generation and expenditure of heat, the result must be very different from that of the first hypothesis. But those who hold this view ought not simply to assert that it may be so, but to show *why* it must.

489. The application above given of the Thermodynamic method to the 10-inch gun shows that it gives results which practically agree with experiment, and the inference is, that although the process of combustion may vary, the effect is practically the same.

490. This is indeed distinctly stated by Count de St. Robert, who says ('Principes de Thermodynamique,' p. 252, Turin, 1870), "Quel que soit la mode de combustion de la charge de poudre dans l'arme à feu, qu'elle se consume instantanément ou successivement, les deux températures t_0, t seront toujours les mêmes. La première dépend de la composition de la poudre; elle est déterminée par la réaction chimique qui s'opère pendant que la poudre passe à l'état gazeux. La seconde ne dépend que du rapport de l'espace, occupé par les gaz lorsqu'ils ont la température t_0 à l'espace qu'ils occupent après la détente dans l'âme derrière

le projectile, et de la pression du milieu extérieur, quantités qui restent invariables."

491. A very striking feature in the summary given above is the large amount of work done in giving energy to and overcoming the friction of the gases. Taking the coefficient of friction so low as ·005, the work done in overcoming it is 1900 foot-tons.

As has been seen, when treating on pressure curves, the coefficient of friction may possibly be higher, and it follows that the amount of friction has a very important bearing on the question of longitudinal strain between the trunnions and the muzzle of the gun.

492. The pressure curves also appear to confirm the hypotheses above adopted respecting the combustion of the charge.

CHAPTER VI.

Concluding Remarks.

493. Up to the present time powder composed of nitrate of potassa, charcoal, and sulphur has been almost exclusively used for artillery, and the proportions of these ingredients have not varied much, but a very great alteration has taken place in the size and form of the grains. These latter have very largely increased, with the view of reducing the pressure, and this object has been still further attained by the Prismatic form, whereby the Ignition is made more gradual.

494. Whilst, in this way, the pressure has become reduced, it has been necessary largely to increase the charges relatively to the weight of projectile, and this, again, has necessitated the increased length of guns. It has further led to increased erosion and other inconveniences.

495. It may very well be asked whether too much has not been sacrificed to this reduction of Pressure, and whether our Ordnance Department has been, and is at the present time, on the right track in gun construction.

496. It may be asked, Where is the necessity of limiting the maximum pressure in a gun to 16 or 17 tons per square inch?

497. In a gun made solid, and of homogeneous metal, there is indeed a limitation to the internal pressure that it will bear, viz. the tensile force of the metal; but in a built-up gun this limitation is not necessary, and it is quite possible to make a gun which will not be strained to more than 20 tons, whilst the internal pressure may be 40 tons per square inch. If, then, the elastic limit of the material of this gun be 25 tons, it is perfectly certain that it may be subjected to an internal pressure of 40 tons without injury. Why, then,

limit the internal pressure to 17 tons, thereby involving a larger charge and many other inconveniences?

498. It is perfectly certain, as appears from M. Sarrau's investigations, fully confirmed by experimental results, that the ballistic effect of a given weight of powder increases as the maximum pressure increases, and therefore the object of the gun constructor should be to increase the strength of the gun, so as to master the force of the powder, rather than to seek for a weak powder to suit a weak gun, and increase the ballistic effect by an increased weight of charge, and length of gun.

This is the path entered on a few years ago, and still persisted in by our gun manufacturers, and it is by this principle that our Ordnance Department is now guided, and our new armament is being constructed.

499. Nothing has been said in the preceding chapters about the new powders which have been invented in France and in this country, because everything connected with them is kept a profound secret, excepting that from time to time we are startled with results said to be obtained with these new explosives.

500. On a recent occasion Lord Armstrong, presiding at a meeting of the Elswick Company, is reported to have said that "with the powder now made by the Chilworth Company, a charge of one-third less weight than hitherto used gives a muzzle velocity of 2400 feet per second"; that is to say, a higher velocity than the ordinary powder. He said nothing about the maximum pressure, but as this is limited by the Ordnance Department to 17 tons per square inch, it is to be presumed that this was not exceeded. Now, it is certain that no such effect can be produced with a charcoal and nitrate powder. The force, or strength, of a powder is, as shown in (§ 108) denoted by $\frac{p_0 v_0 \mathrm{T}_0}{273}$, which is nearly constant for all such powders; but it is quite possible that the product of the two variables v_0 and T_0 may be increased by the use of other ingredients. If, for instance, some ingredient other

than charcoal were used, capable of giving an increased volume of gas with the same value of T_0, a more powerful powder would result. If, again, v_0 remains constant, whilst T_0 was increased, the same result would ensue, and *à fortiori*, if both v_0 and T_0 were increased, a still more powerful powder would be obtained.

501. The question, however, still remains, What will be the effect of such powder as regards the maximum pressure, the erosion of the gun, and the storage and keeping properties of such powder?

502. "Stability" of constitution is of the utmost importance in gunpowder. If it be liable to change so as to alter its "characteristics," it is evident that range tables must become of little use.

503. Such change may be brought about either by a change in the hygroscopic or in the chemical condition of the powder.

504. Charcoal powders are only subject to the first of these changes, and the less moisture they contain the more stable they are. For this reason it may be expected that cocoa powder, which contains a high percentage of moisture, will be more subject to change and become more violent in hot climates than the black powders, which contain less moisture.

505. Charcoal powders are not subject to chemical change except at very high temperatures. On the other hand, powders composed of substances which have greater chemical affinity *inter se*, may be expected to undergo considerable change when kept under certain conditions differing from those under which they were manufactured.

506. Although the composition of the new powders is kept secret, it is very probable that they are to a great extent compounds of ammonia and picrates or their analogues, and if so they are no doubt liable to change if kept for a time in a moderately high temperature, such as would obtain on board ship or in magazines in the tropics.

507. Such change might not only affect the chemical con-

stitution but might possibly destroy to some extent the cohesion of the grains, leading to their disintegration or fracture, and in this case a very violent action might ensue, giving rise to high and dangerous abnormal pressures.

508. It is therefore of great importance that the stability of "characteristic" of these new powders should be completely investigated, and as these important questions can only be definitely settled by long experience, it would be very rash to adopt such powders into the Service until such experience has been obtained.

POSTSCRIPT.

AFTER the preceding pages had gone to press, I received, through the kindness of my friend Lieut. Crozier, copies of 'Notes on the Construction of Ordnance,' Nos. 36 and 42. No. 36 is by Lieut. W. M. Medcalfe, of the Ordnance Department U.S. Army, and is dated 28th April, 1886. It is entitled 'Application of Sarrau's Formulas to American Powders and Guns.'

No. 42 is a translation of Sarrau's 'Recherches Théoriques sur le Chargement des Bouches à feu,' with Notes and Appendix by Lieut. D. A. Howard, Ordnance Department U.S. Army, and is dated 19th August, 1887.

The publication of these documents for the use of officers of the Ordnance Department, by authority of the Secretary of State for War of the United States, is evidence of the high value attached in that country to M. Sarrau's investigations, the practical value of which is evident by the tables which are reproduced at the end of this Postscript, and which will be found of great interest to artillery officers.

Table I. contains the ballistic elements of various American service guns, with their relative powders and the ballistic results obtained.

Table II. gives the "characteristics" of sixteen American powders.

Table III. shows the verification of the "characteristics" by comparison of the actual muzzle relations and pressures with those deduced from the formulæ.

Table A, taken from Lieut. Howard's Appendix to No. 42, gives the "Characteristics" of a number of brown prismatic powders tested in an 8-inch gun.

Table B. The verification of some of these same powders in the same gun.

An examination of these tables, which give the results of 138 rounds fired from guns varying from 3·2-inch to 12-inch calibre, and with different powders, shows how satisfactorily M. Sarrau's method represents the actual facts of artillery practice, and consequently how important his investigations are, both as regards ballistic practice and the construction of guns.

Lieut. Medcalfe, in paper No. 36, points out that the value of the "force" of the powder is not constant, especially as regards the brown prismatic powders, and in the Table II. it will be seen that it varies from 1·096 to 0·735 in cocoa powder.

The probability of this variation is pointed out in (§ 327) page 153 *ante*, and the value of f for cocoa powder was there estimated at 0·7635.

The value of f for cocoa powder must, however, be taken with much reserve. It is well known that with this powder, and especially when fired in large charges, some of the grains are blown out only partially consumed, and of such a case M. Sarrau's formula does not take account; and the value of f determined from any particular experiment must be considered as only an approximate value, and correct only as regards the ballistic elements of that experiment.

It should be the object of the artillerist to have such a value of τ as will ensure the complete combustion of every grain before the projectile leaves the gun.

Lieut. Medcalfe observes in "Notes" 36, page 10, that—"Any change in the method of manufacture which favours the regularity of burning of the grains will increase the effective strength, and enable us to obtain with the same maximum pressure higher velocities.

"Assuming $f = 1$ as a mean value of the powder that is easily obtainable, and giving for ordinary densities and pressures values of τ small enough to obtain the complete combustion of the grains before the projectile leaves the gun,

we will attempt to deduce the relation between the density and τ for brown prismatic powder.

"N V_1, Table II. may be taken as the standard, and the value of K in M. Sarrau's equation $\tau = K\left(\frac{e}{1 \cdot 875 - \delta}\right)$ (§ 243) page 119 *ante*, making

$$\epsilon = 0 \cdot 475 \text{ inches} = 0 \cdot 121 \text{ dm.}$$
$$\delta = 1 \cdot 828$$

gives

$$K = 1 \cdot 217."$$

Lieut. Medcalfe shows, No. 36, page 9, that with the 12-inch gun the best result is obtained with a charge of 246 lb. of powder, and a value of $\tau = 1 \cdot 593$ (see § 307, page 144 *ante*, and No. 36, page 9), and using the value of $K = 1 \cdot 217$ just obtained, we find $\delta = 1 \cdot 804$, which is nearly that of N.R., Table II.

Referring to M. Sarrau's formula (15) (§ 174) and (42*a*) (§ 207), Lieut. Medcalfe observes that with slow powders the velocity varies as $\frac{f^{\frac{1}{2}} a^{\frac{1}{2}}}{\tau^{\frac{1}{4}}}$, and the pressure as $\frac{f a}{\tau}$, and consequently if f and a remain constant, any increase of τ which reduces the pressure must also reduce the velocity, though in a less proportion. If, however, we wish to decrease the pressure and retain the velocity unchanged, f and τ must be both increased in such a manner that $\frac{f}{\tau}$ shall decrease, whilst $\frac{f^{\frac{1}{2}}}{\tau^{\frac{1}{4}}}$ remains constant.

Taking, for instance—

Cocoa powder $\tau = 1 \cdot 130$ and $f = \cdot 735$, and N.V. powder $\tau = 1 \cdot 952$, and $f = \cdot 968$, we get $\frac{f^{\frac{1}{2}}}{\tau^{\frac{1}{4}}} = \cdot 832$ for both, and $\frac{f}{\tau} = \cdot 650$ for cocoa, and $= \cdot 496$ for N.V. Consequently,

the velocity is the same, whilst the pressure is decreased in the above proportion.

From this it follows that any increase in f is of great importance, and the efforts of the manufacturers should be directed to obtain this increase; and it is in this direction that powder manufacturers are now moving, partly by an alteration in the constituents of the powder, and partly by varying the mechanical process of manufacture.

TABLE I.

Powder.	Gun.	e inches.	l inches.	p pounds.	π pounds.	Δ	V feet per sec.	P_0 lbs. per sq. in.
L X ..	3″·20 B. L. rifle	3·2	73·2	13	3·50	0·857	1,649	31,000
L X B ..	Ditto	3·2	73·2	13	3·75	0·827	1,756	35,150
I K D ..	Ditto	3·2	73·2	13	3·50	0·857	1,630	29,100
I K B ..	Ditto	3·2	73·2	13	3·50	0·857	1,663	30,500
K H C ..	12″ mortar	12·0	91·6	610	50·0	0·821	932	22,000
M W ..	Ditto	12·0	91·6	610	48·0	0·788	959	26,250
E V F ..	8″ B. L. rifle	8·0	98·5	183	45·0	0·792	1,488	32,650
N M ..	12″ B. L. rifle	12·0	273·5	800	265·0	0·827	1,688	26,350
N V_3 ..	Ditto	12·0	273·5	800	265·0	0·827	1,718	26,890
N R ..	Ditto	12·0	273·5	800	265·0	0·827	1,826	32,990
N V_1 ..	Ditto	12·0	273·5	800	265·0	0·827	1,760	26,625
N V_2 ..	Ditto	12·0	273·5	800	265·0	0·827	1,756	28,000
I B ..	3″·17 M. L. rifle	3·175	74·6	10·5	5·469	0·814	1,933	25,000
O B ..	12″ mortar	12·0	91·6	610	52	0·854	987	25,250
O C ..	Ditto	12·0	91·6	610	52	0·854	942	19,750

TABLE II.—CHARACTERISTICS OF DIFFERENT POWDERS.

Powder.	Form of grain.	e inches.	N.	Sp. gr.	x.	a.	λ.	τ.	f.	Log α.	Log β.	Log a^2.
L X ..	Square prism	0·315	270	1·706	0·720	2·442	0·803	0·552	0·967	0·31562	0·16297	0·63124
L X B ..	Ditto	0·315	270	1·706	0·720	2·442	0·803	0·540	1·055	0·33940	0·17243	0·67880
I K D ..	Granulated	..	2,200	1·725	..	3·000	1·000	0·727	0·970	0·30115	0·13851	0·60230
I K B ..	Ditto	..	2,200	1·725	..	3·000	1·000	0·714	1·001	0·31273	0·14618	0·62546
I B ..	Sphero-hexagonal	..	123	1·728	..	3·000	1·000	1·183	1·064	0·21561	$\bar{1}$·92707	0·43122
K H C	Ditto	..	123	1·775	..	3·000	1·000	1·267	0·979	0·18257	$\bar{1}$·89721	0·36514
O B ..	Hexagonal	..	123	1·725	..	3·000	1·000	1·269	1·050	0·19747	$\bar{1}$·89671	0·39493
O C ..	Ditto	..	123	1·750	..	3·000	1·000	1·565	1·013	0·14412	$\bar{1}$·81541	0·28824
M W ..	Ditto	..	72	1·725	..	3·000	1·000	1·106	1·096	0·23644	$\bar{1}$·95615	0·47288
E V F ..	Ditto	..	72	1·750	..	3·000	1·000	1·351	1·043	0·18232	$\bar{1}$·86937	0·36464
Cocoa ..	Pierced hexagonal prism	0·475	11½	1·863	0·500	1·500	0·333	1·130	0·735	$\bar{1}$·95922	$\bar{1}$·39892	$\bar{1}$·91844
N M ..	Ditto	0·475	11½	1·833	0·500	1·500	0·333	1·589	0·782	$\bar{1}$·93402	$\bar{1}$·32182	$\bar{1}$·86805
N R ..	Ditto	0·475	11½	1·814	0·500	1·500	0·333	1·363	0·839	$\bar{1}$·98282	$\bar{1}$·38846	$\bar{1}$·96565
N V 1 ..	Ditto	0·475	11½	1·830	0·500	1·500	0·333	1·952	0·968	$\bar{1}$·93628	$\bar{1}$·23245	$\bar{1}$·87256
N V 2 ..	Ditto	0·475	11½	1·818	0·500	1·500	0·333	1·665	0·870	1·94721	1·30149	1·89443
N V 3 ..	Ditto	0·475	11½	1·826	0·500	1·500	0·333	1·694	0·851	$\bar{1}$·93843	1·29384	$\bar{1}$·87686

TABLE III.—VERIFICATION OF THE CHARACTERISTICS DEDUCED FOR THE DIFFERENT POWDERS.

Gun.	Kind of powder.	c.	l.	p.	π.	Δ.	Velocities.		Pressures.		Number of rounds considered.
							Calculated from binomial formula.	Measured.	Calculated pounds per sq. inch.	Measured pounds per sq. inch.	
							feet.	feet.			
12-inch B. L. rifle	Cocoa	12	273·5	750	265	0·827	1,761	1,761	29,051	32,000	1
12¾-inch M. L. rifle ..	Ditto	12·25	183·4	693	175	·999	1,460	1,460	24,404	23,800	1
12-inch M. L. mortar ..	M W	12	91·6	610	26	·427	652	651	8,978	9,500	3
Ditto	Ditto	12	91·6	610	45	·739	919	925	23,477	24,750	1
Ditto	Ditto	12	91·6	610	48	·788	957	959	26,250	26,250	1
8-inch B. L. rifle	E V F	8	98·5	183	45	·792	1,488	1,488	32,650	32,650	5
Ditto	Ditto	8	98·5	183	50	·872	1,585	1,581	38,870	38,325	2
12-inch M. L. mortar ..	Ditto	12	91·6	610	52	·854	979	980	23,549	25,000	2
Ditto	K H C	12	91·6	610	50	·821	932	932	22,000	21,750	1
Ditto	Ditto	12	91·6	610	52	·854	953	993	23,574	25,000	2
8-inch M. L. rifle	Ditto	8	95	180	35	·9	1,368	1,388	31,226	31,100	10
3·2-inch B. L. rifle, steel	I K D	3·2	73·2	13	3·5	·857*	1,630	1,630	29,000	29,100	25
Ditto	Ditto	3·2	73·2	13½	3·5	·857	1,603	1,593	29,275	29,000	3
Ditto	Ditto	3·2	73·2	15	3·5	·857	1,530	1,529	30,057	30,700	1
Ditto	I K B	3·2	73·2	13	3·5	·857	1,663	1,663	30,590	30,590	3
Ditto	Ditto	3·2	73·2	13	3·75	·827†	1,692	1,702	30,958	29,770	12
3·2-inch B. L. converted	Ditto	3·2	56·6	13	3·0	·903	1,510	1,501	28,700	27,700	5
47mm torpedo	Ditto	1·85	61·54	3·281	1·152	·910	1,802	1,828	..	..	2
3·2-inch B. L. rifle, steel	L X B	3·2	73·2	13	3·75	·827	1,756	1,756	35,150	35,150	8
Ditto	Ditto	3·2	73·2	13	3·5	·857	1,726	1,713	34,587	36,500	8
47mm torpedo	Ditto	1·85	61·54	3·281	1·063	·840	1,805	1,773	..	..	12
3·17-inch M. L. rifle ..	I B	3·175	74·6	10·5	5·469	·814	1,933	1,933	25,000	24,500	2
Ditto	Ditto	3·175	74·6	10·5	5·625	·837	1,967	1,965	26,261	26,750	2
Ditto	Ditto	3·175	74·6	10·5	5·818	·865	2,008	2,013	27,812	30,000	2

* De Bange gas-check used.

† Freyre gas-check used.

TABLE A.—CHARACTERISTICS OF EXPERIMENTAL BROWN PRISMATIC POWDERS, TESTED IN THE 8-INCH B.L. STEEL RIFLE.

Designation of the Powder.	log α^2.	log α.	log β.	N.	Sp. gr.	f.	τ.
Slow powders that gave high velocities and moderate pressures.							
P N A ..	$\bar{1}$·93792	$\bar{1}$·96896	$\bar{1}$·28041	10·73	1·840	1·009	1·748
P N ..	$\bar{1}$·97407	$\bar{1}$·98704	$\bar{1}$·33725	10·85	1·840	0·963	1·533
German cocoa	$\bar{1}$·96950	$\bar{1}$·98475	$\bar{1}$·32061	11·50	1·858	0·958	1·541
P O ..	$\bar{1}$·95401	$\bar{1}$·97701	$\bar{1}$·28978	10·71	1·843	1·026	1·710
Medium powders that gave good results.							
Q W ..	0·01571	0·00786	$\bar{1}$·37924	10·65	1·816	0·962	1·392
Q W A ..	0·01738	0·00869	$\bar{1}$·39508	10·66	1·822	0·931	1·342
Q Y ..	0·00579	0·00290	$\bar{1}$·36094	10·67	1·817	0·981	1·452
Q M ..	$\bar{1}$·96891	$\bar{1}$·98446	$\bar{1}$·37118	10·87	1·834	0·880	1·418
Slow powders that gave unsatisfactory velocities.							
Q P ..	$\bar{1}$·93139	$\bar{1}$·96570	$\bar{1}$·29918	10·85	1·837	0·953	1·674
Q Q ..	$\bar{1}$·90367	$\bar{1}$·95184	$\bar{1}$·30009	10·77	1·849	0·892	1·670
A medium powder that gave unsatisfactory results.							
Q N ..	0·01204	0·00602	$\bar{1}$·39077	10·76	1·825	0·929	1·355
Powders entirely too quick.							
Q V ..	0·02529	0·01265	$\bar{1}$·40456	10·68	1·825	0·928	1·313
Q U ..	0·01987	0·00998	$\bar{1}$·40556	10·71	1·840	0·914	1·310
P A ..	0·01968	0·00984	$\bar{1}$·44259	10·80	1·825	0·839	1·203
Q W B ..	0·05348	0·02674	$\bar{1}$·45039	10·66	1·834	0·891	1·182
Q X ..	0·06018	0·03009	$\bar{1}$·45585	10·68	1·825	0·894	1·167
Q Y A ..	0·06359	0·03180	$\bar{1}$·48169	10·65	1·826	0·849	1·099
Q K ..	0·06041	0·03020	$\bar{1}$·48208	10·68	1·828	0·842	1·098
Q L ..	0·18085	0·09043	$\bar{1}$·55701	10·79	1·825	0·935	0·924
Powders entirely too slow.							
Q E ..	$\bar{1}$·87038	$\bar{1}$·93519	$\bar{1}$·15363	10·76	1·840	1·157	2·340
Q H ..	$\bar{1}$·73814	$\bar{1}$·86907	$\bar{2}$·65295	10·75	1·840	2·704	7·412

TABLE B.

VERIFICATION OF THE CHARACTERISTICS OF SOME OF THE EXPERIMENTAL POWDERS FOR THE 8-INCH B.L. STEEL RIFLE.

Designation of the Powder.	π.	p.	Velocities.		Pressures.		No. of Rounds Considered.
			Measured.	Computed.	Measured.	Computed.	
			feet per second.	feet per second.	pds. per sq. in.	pds. per sq. in.	
German cocoa	110	289	1,875	1,879	35,900	35,623	1
German cocoa	110	302	1,857	1,871	37,112	36,016	1
P N A	113	301	1,852	1,842	35,400	35,077	2
P N	105	289	1,825	1,824	33,662	33,186	1
P N	110	289	1,878	1,878	36,000	36,000	1
Q U	110	300	1,876	1,903	40,700	40,378	1
Q U	105	289	1,860	1,873	35,640	36,877	1
Q U	100	300	1,794	1,794	34,255	34,255	1
Q V	105	289	1,878	1,886	37,340	37,600	1
P O	105	289	1,818	1,811	31,700	31,688	1
P O	110	289	1,864	1,864	34,400	34,400	1
P A	100	235	1,940	1,921	32,950	32,138	1
P A	100	286	1,820	1,802	35,400	34,734	1
P A	100	289	1,788	1,787	34,150	33,844	1
P N A	113	289	1,857	1,862	34,350	34,679	1
P N A	105	289	1,771	1,782	30,900	30,535	1
Q Y A	95	289	1,808	1,797	35,500	34,230	2
Q W A	105	289	1,881	1,876	36,500	36,660	1
Q E	110	301	1,746	1,733	30,200	28,643	1
Q E	113	300	1,767	1,764	29,825	30,000	1
Q Y	105	289	1,904	1,877	34,600	35,690	1
Q K	100	300	1,815	1,812	36,605	37,520	1

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